AMERICAN MENTAL ARITHMETIC

BAILEY

AMERICAN · BOOK · COMPANY
NEW YORK · CINCINNATI · CHICAGO

PREFACE.

In the solution of problems there are two distinct steps — the selection of the operations, and their performance. Mental and written arithmetic agree in that the choice of operations is determined in the same manner; they differ in that the operations are wholly mental in the one, while external aids are used in the other. Mental arithmetic should, therefore, embrace all cases in written arithmetic except those which teach how to add, subtract, multiply, and divide large numbers. This arithmetic is intended as a drill-book in which the principles of written arithmetic, except those mentioned above, shall be concisely stated and illustrated. The examples and problems are such as the average mind should be able to solve readily without a pencil.

He who teaches by the printed page must use every artifice of arrangement to make his statements clear and attractive. The placing of principles and illustrations in parallel columns aids the student to grasp the subject as a whole, since each column may be read independently, and each conveys the same thought in a different manner. The beginning of each subject at the top of a page, the systematic placing of explanations and directions under exercises, and the continuous numbering of all the examples in a chapter, aid the teacher to *announce* and the pupil to *understand* the requirements.

In Addition, the combination method is made prominent. The number of seconds which should be required for the solution of each example is stated after each exercise. Since ninety per cent of all arithmetical computation in the work-shop, farm, and counting-room is Addition, this subject cannot be too zealously pressed. Many who have broken the habit, in adding, of saying "6 and 8 are 14 and 6 are 20," are still saying in subtracting, "6 from 10 leaves 4"; in multiplying, "9 times 8 are 72, and 4 are 76"; and in dividing, "$12 \div 5 = 2$ and 2 remaining." Special

stress is laid upon the importance, in performing operations, of dropping all unnecessary words, since the mind reaches results much more rapidly without them.

In factoring, the introduction of a new conception, that of *numbers severally prime* to each other, will be appreciated by experts, and cannot fail to benefit learners, because it obviates the cumbersome expression of numbers by their prime factors. Those who, in dividing fractions, have never practiced mentally the method largely used in Europe, will be delighted with the ease by which results can be obtained.

Attention is called to the presentation of the Metric System. By memorizing the table of submultiples and the table of units, the student acquires the principles of the whole subject, and will only need practice to master it.

Percentage is taught without rules or formulæ, and without the use of the terms *base*, *amount*, and *difference*, although one page is devoted to them after the subject has been completed. The student comes to see clearly that the various exercises in percentage do not need special rules, but are familiar cases slightly modified since the symbol "%" is used instead of *hundredths*. Interest is taught by the 6 % method and by the modification of this method in general use among bankers.

The practical exercises "at the lumber yard," "at the carpet store," etc., are to drill the student in methods daily used at such places. Mensuration has been developed with a view of showing the necessity for the existence of the various forms, their relations, and their limitations.

Few principles are presented, but these few are the keys to all departments of the science. Let it be remembered, that he who relies upon thousands of special rules is but a pygmy beside the giant who can apply a score of general principles to millions of particulars.

M. A. BAILEY.

State Normal School of Kansas.

TABLE OF CONTENTS.

	PAGE
ADDITION	7
Combinations — Two Figures	8
Combinations — Three Figures	14
Combinations — Four Figures	15
Addends	16
Problems	18
SUBTRACTION	21
Problems	26
MULTIPLICATION	29
Problems	33
DIVISION	36
Precedence of Signs	41
Parenthesis or Bar	42
Problems	45
FACTORING	48
Multiplication and Division	52
Greatest Common Divisor	54
Least Common Multiple	55
COMMON FRACTIONS	56
First Conception — An Expression of Division	56
Second Conception — One or More of the Equal Parts of a Unit	57
Change of Form — To Higher Terms	58
Change of Form — To Lower Terms	59
Change of Form — To a Whole or Mixed Number	60
Addition and Subtraction	61
Multiplication — Universal Case	62
Division — Universal Case	63
Problems	64
DECIMALS	72
Reduction — Common Fractions to Decimals	74
Reduction — Decimals to Common Fractions	75
Per Cent	76
Short Methods	77
DENOMINATE NUMBERS — ENGLISH TABLES	81
Money, Weights	81
Long Measure	82

PAGE

DENOMINATE NUMBERS — ENGLISH TABLES (*Continued*).
Square and Cubic Measures 83
Capacity, Time 84
Circular Measure, Counting, Paper, Equivalents 86
Exercises in Tables 87
Reduction in Same Table 90
Reduction Table to Table 92
DENOMINATE NUMBERS — METRIC SYSTEM 94
Practical Questions 98
Reduction — Metric 99
Reduction — English and Metric 100
PERCENTAGE 101
Reduction 101
The Operation Directly Stated 102
Operations to be Determined 104
Profit and Loss 109
Commission 112
INTEREST 115
Simple Interest 116
Trade Discount 123
True Discount 124
Bank Discount 125
Stocks 126
PRACTICAL EXERCISES 131
At the Lumber Yard 131
Measurement of Logs 135
At the Carpet Store 136
With the Paper Hanger 137
Average 138
INVOLUTION AND EVOLUTION 139
PROPORTION 140
MENSURATION 141
One Dimension 141
Two Dimensions 142
Three Dimensions 147
Similarity 150
MISCELLANEOUS 152
Arithmetical Progression 152
Geometrical Progression 153
Specific Gravity 154
Zero and Infinity 155
GENERAL REVIEW EXERCISES 156

AMERICAN MENTAL ARITHMETIC.

ADDITION.

Addition is indicated by the sign +.

The numbers to be united are *addends;* the result, the *sum* or *amount.*

The sign of equality is =.

The sum of two or more numbers may be found by counting.

Addition is a process shorter than counting for finding the sum of numbers.

A number may be written by the decimal notation or by its addends.

A number may be spelled by naming its addends, just as a word is spelled by naming its letters.

A number may be spelled in several different ways.

ILLUSTRATION.

$6 + 4 = 10$

read

6 plus 4 equals 10.

6 and 4, *addends.*

10, *sum* or *amount.*

To find the sum of 6 *and* 4 *by counting.*

Counting to 6 and making a mark at each count, //////; counting to 4 and making a mark at each count, ////// ////; counting the result, we have 10.

Ten may be written

10; or $\overset{5}{5'}\ \overset{6}{4'}\ \overset{7}{3'}\ \overset{8}{2'}\ \overset{9}{1'}$

Ten, as written above, is spelled *five five*, *six four*, *seven three*, *eight two*, or *nine one.*

§ 1. Combinations — Two Figures.

There are 45 different combinations of figures taking two at a time, viz.:

9 9 9 9 9 9 9 9 9 8 8 8 8 8 8 8 8 7 7 7 7 7 7
9' 8' 7' 6' 5' 4' 3' 2' 1' 8' 7' 6' 5' 4' 3' 2' 1' 7' 6' 5' 4' 3' 2'

7 6 6 6 6 6 6 5 5 5 5 5 4 4 4 4 3 3 3 2 2 1
1' 6' 5' 4' 3' 2' 1' 5' 4' 3' 2' 1' 4' 3' 2' 1' 3' 2' 1' 2' 1' 1'

These combinations are shown in the following table, and should be thoroughly memorized.

The Number.	Combinations.	The Number.	Combinations.	The Number.	Combinations.
2	1 1	8	4 5 6 7 4' 3' 2' 1	14	7 8 9 7' 6' 5
3	2 1	9	5 6 7 8 4' 3' 2' 1	15	8 9 7' 6
4	2 3 2' 1	10	5 6 7 8 9 5' 4' 3' 2' 1	16	8 9 8' 7
5	3 4 2' 1	11	6 7 8 9 5' 4' 3' 2	17	9 8
6	3 4 5 3' 2' 1	12	6 7 8 9 6' 5' 4' 3	18	9 9
7	4 5 6 3' 2' 1	13	7 8 9 6' 5' 4		

Memorize thus: 2, $\frac{1}{1}$ (*one one*); 3, $\frac{2}{1}$ (*two one*); 4, $\frac{2}{2}$, $\frac{3}{1}$ (*two two*, or, *three one*); etc.

1. What are the combinations whose sum is 10?
2. What are the combinations whose sum is 12? 18? 9? 4?
3. What are the combinations whose sum is 17? 5? 11? 3?
4. What are the combinations whose sum is 2? 16? 6? 15?
5. What are the combinations whose sum is 7? 14? 8? 13?

State the answers without reading the questions aloud.

Ex. 1. 5 6 7 8 9
5' 4' 3' 2' 1

Name the combinations whose sum is:

6. Two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen.

7. Eighteen, seventeen, sixteen, fifteen, fourteen, thirteen, twelve, eleven, ten, nine, eight, seven, six, five, four, three, two.

8. Ten, twelve, fifteen, two, six, eleven, nine, sixteen, three, thirteen, four, seventeen, five, eighteen, seven, fourteen, eight.

State the answers without reading the questions aloud.

Ex. **7.** $\frac{9}{9}$; $\frac{9}{8}$; $\frac{8}{8'}$ $\frac{9}{7}$; $\frac{8}{7'}$ $\frac{9}{6}$; etc.

Read the sums:

9.	9	7	8	4	7	6	7	8	6	3	2	4	6	7
	9'	6'	5'	3'	2'	5'	8'	8'	4'	7'	8'	4'	9'	6'
10.	5	6	9	7	2	4	8	4	4	5	8	3	7	7
	5'	3'	2'	2'	6'	4'	8'	1'	3'	8'	7'	3'	4'	5'
11.	4	4	3	8	9	6	1	2	7	9	4	6	2	4
	8'	6'	5'	9'	9'	8'	1'	1'	7'	5'	9'	7'	2'	2'

Ex. **9.** 18, 13, 13, 7, 9, etc. Do not say 9 and 9 are 18.

Read the sums:

12. 9+9, 3+2, 6+6, 2+2, 5+5, 4+3, 7+6, 9+8, 6+5, 2+1, 8+8, 7+7, 1+1, 5+4, 8+7, 4+4, 3+3.

13. 9+7, 6+2, 9+4, 8+6, 9+6, 7+2, 6+4, 5+1, 9+5, 7+5, 8+4, 5+3, 8+5, 5+2, 7+1, 8+3, 7+3, 8+2, 6+3, 6+1, 4+2, 9+3, 7+4, 4+1.

14. 8+9, 7+6, 4+5, 2+3, 7+8, 8+4, 7+9, 7+7, 8+4, 7+2, 8+7, 9+9, 8+8.

Do not say 9 plus 9 equals 18, but state sums directly.

Ex. **12.** 18, 5, 12, etc.

To 1, 3, 6, 8, 9, 4, 7, 5, 2

15. Add 9.	**18.** Add 6.	**21.** Add 3.
16. Add 8.	**19.** Add 5.	**22.** Add 2.
17. Add 7.	**20.** Add 4.	**23.** Add 1.

Ex. **15.** 10, 12, 15, 17, 18, etc.

State the results:

24. 8+9=	**32.** 6+9=	**40.** 9+5=
25. 7+8=	**33.** 4+7=	**41.** 3+9=
26. 8+4=	**34.** 6+8=	**42.** 9+2=
27. 6+5=	**35.** 9+9=	**43.** 6+6=
28. 5+9=	**36.** 6+7=	**44.** 8+8=
29. 3+7=	**37.** 7+5=	**45.** 4+9=
30. 6+3=	**38.** 8+5=	**46.** 3+8=
31. 4+6=	**39.** 7+9=	**47.** 7+7=

48. How many are 9 and 7? 6 and 5? 4 and 9? 3 and 7? 5 and 8? 7 and 2? 8 and 6?

49. How many are 7 and 6? 8 and 3? 9 and 5? 8 and 8? 6 and 2? 4 and 5? 5 and 3? 7 and 9? 8 and 8? 4 and 9? 4 and 3?

50. How many are 9 and 9? 6 and 4? 5 and 7? 8 and 9? 3 and 6? 5 and 2? 9 and 6? 8 and 7? 7 and 7? 6 and 6? 5 and 5?

51. Read as rapidly as possible 12, 16, 14, 13, 11, 18, 14, 16, 17.

52. Read as rapidly as possible

6	9	8	7	5	9	9	8	9
6'	7'	6'	6'	6'	9'	5'	8'	8'

The student has mastered these combinations when he can read numbers as expressed in Ex. **52** as rapidly as he can read numbers as expressed in Ex. **51.**

§ 2. In General.

Declare the sums:

53. $\begin{matrix} 8 & 28 & 38 & 58 & 78 & 88 \\ 9' & 9' & 9' & 9' & 9' & 9 \end{matrix}$.

54. $\begin{matrix} 5 & 35 & 55 & 85 & 65 & 25 \\ 7' & 7' & 7' & 7' & 7' & 7 \end{matrix}$.

55. $\begin{matrix} 9 & 89 & 29 & 79 & 39 & 69 \\ 9' & 9' & 9' & 9' & 9' & 9 \end{matrix}$.

56. $\begin{matrix} 8 & 48 & 68 & 98 & 78 & 58 \\ 6' & 6' & 6' & 6' & 6' & 6 \end{matrix}$.

57. $\begin{matrix} 9 & 89 & 39 & 49 & 59 & 79 \\ 4' & 4' & 4' & 4' & 4' & 4 \end{matrix}$.

58. $\begin{matrix} 8 & 48 & 88 & 78 & 58 & 28 \\ 8' & 8' & 8' & 8' & 8' & 8 \end{matrix}$.

Ex. **53.** 17, 37, 47, etc.

What is the right hand figure of the sum,

59. When 9 is added to a number ending in 9? 8? 3? 7? 4? 2? 5? 6? 1?

60. When 8 is added to a number ending in 8? 2? 6? 3? 5? 9? 1? 4? 7?

61. When 7 is added to a number ending in 1? 3? 5? 7? 9? 2? 4? 6? 8?

62. When 6 is added to a number ending in 6? 3? 8? 2? 4? 1? 5? 7? 9? When 5 is added?

63. When 4 is added to a number ending in 2? 6? 4? 8? 1? 9? 5? 7? 3? When 3 is added?

64. When 2 is added to a number ending in 3? 5? 1? 7? 4? 9? 2? 6? 8? When 1 is added?

Ex. **59.** 8, 7, 2, 6, 3, 1, 4, 5, 0.

Beginning with 1, count as rapidly as possible to about 100:

65. By 9.
66. By 8.
67. By 7.
68. By 6.
69. By 5.
70. By 4.
71. By 3.
72. By 2.
73. By 1.

Ex. **65.** 1, 10, 19, 28, 37, etc.

Add:

74. 3, 7, 6, 8, 9, 2, 7, 8, 5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 5, 9, 3.
75. 4, 9, 8, 1, 1, 3, 9, 9, 7, 3, 4, 5, 6, 7, 8, 9, 6, 8, 7, 8, 5.
76. 6, 8, 2, 4, 1, 3, 5, 7, 9, 1, 4, 7, 2, 6, 1, 6, 8, 9, 9, 8, 6.
77. 7, 8, 9, 9, 8, 7, 6, 5, 4, 4, 5, 6, 3, 8, 7, 7, 8, 3, 7, 4, 7.
78. 5, 3, 7, 6, 2, 4, 9, 8, 7, 4, 5, 9, 2, 9, 8, 7, 8, 6, 3, 8, 9.
79. 2, 7, 5, 3, 9, 6, 8, 8, 8, 5, 6, 2, 5, 6, 8, 3, 4, 7, 6, 6, 9.
80. 8, 7, 7, 6, 6, 5, 5, 4, 3, 2, 1, 6, 5, 4, 3, 3, 5, 7, 9, 8, 7.
81. 9, 6, 3, 4, 2, 4, 6, 8, 9, 8, 7, 6, 3, 1, 3, 5, 9, 9, 8, 4, 5.
82. 4, 3, 5, 8, 7, 6, 5, 1, 3, 2, 4, 5, 7, 8, 2, 4, 5, 7, 6, 8, 8.
83. 3, 5, 7, 2, 2, 4, 6, 7, 8, 3, 6, 9, 9, 8, 7, 6, 3, 2, 5, 1, 7.
84. 8, 2, 4, 5, 6, 2, 7, 8, 6, 4, 5, 9, 8, 3, 4, 5, 3, 6, 9, 4, 2.
85. 9, 9, 8, 7, 3, 4, 6, 1, 5, 4, 2, 3, 3, 9, 1, 7, 8, 9, 7, 8, 2.
86. 3, 5, 6, 9, 4, 3, 3, 4, 6, 1, 4, 3, 1, 6, 9, 9, 6, 2, 3, 2, 4.
87. 8, 2, 4, 8, 1, 5, 1, 5, 8, 3, 3, 1, 9, 5, 4, 8, 6, 3, 5, 8, 9.
88. 1, 5, 8, 6, 3, 5, 1, 9, 3, 3, 7, 7, 2, 5, 9, 2, 8, 6, 6, 2, 2.

Ex. **74.** 3, 10, 16, 24, 33, 35, etc. Allow 9 seconds for each example.

89. On the next page find the sum of the columns giving the population of the U. S. in 1850.

90. In the same manner find the sum of the columns for 1860.

91. Find the sum of the columns for 1870.

92. Find the sum of the columns for 1880.

93. Find the sum of the columns for 1890.

Ex. **89.** 9, 16, 20, 25, 31, . . . 166; 5, 13, 22, 24, 28, . . . 171; etc. Allow 19 seconds for a column.

U.S.	1850	1860	1870	1880	1890
N.Y.	3,097,394	3,880,735	4,382,759	5,082,871	5,997,853
Penn.	2,311,786	2,906,215	3,521,951	4,282,891	5,258,014
Ill.	851,470	1,711,951	2,539,891	3,077,871	3,826,351
Ohio	1,980,329	2,339,511	2,665,260	3,198,062	3,672,316
Mo.	682,044	1,182,012	1,721,295	2,168,380	2,679,184
Mass.	994,514	1,231,066	1,457,351	1,783,085	2,238,943
Tex.	212,592	604,215	818,579	1,591,749	2,235,523
Ind.	988,416	1,350,428	1,680,637	1,978,301	2,192,404
Mich.	397,654	749,113	1,184,059	1,636,937	2,063,889
Iowa	192,214	674,913	1,194,020	1,624,615	1,911,896
Ky.	982,405	1,155,684	1,321,011	1,648,690	1,858,635
Ga.	906,185	1,057,286	1,184,109	1,542,180	1,837,353
Tenn.	1,002,717	1,109,801	1,258,520	1,542,359	1,767,518
Wis.	305,391	775,881	1,054,670	1,315,497	1,686,880
Va.	1,421,661	1,596,318	1,225,163	1,512,565	1,655,980
N.C.	869,039	992,622	1,071,361	1,399,750	1,617,947
Ala.	771,623	964,201	996,992	1,262,505	1,513,017
N.J.	489,555	672,035	906,096	1,131,116	1,444,933
Kans.		107,206	364,399	996,096	1,427,096
Minn.	6,077	172,023	439,706	780,773	1,301,826
Miss.	606,526	791,305	827,922	1,131,597	1,286,600
Cal.	92,597	379,994	560,247	864,694	1,208,130
S.C.	668,507	703,708	705,606	995,577	1,151,149
Ark.	209,897	435,450	484,471	802,525	1,128,179
La.	517,762	708,002	726,915	939,946	1,118,587
Nebr.		28,841	122,993	452,402	1,058,910
Md.	583,034	687,049	780,894	934,943	1,042,390
W.Va.			442,014	618,457	762,794
Conn.	370,792	460,147	537,454	622,700	746,258
Me.	583,169	628,279	626,915	648,936	661,086
Colo.		34,277	39,864	194,327	412,198
Fla.	87,445	140,424	187,748	269,493	391,422
N.H.	317,976	326,073	318,300	346,991	376,530
R.I.	147,545	174,620	217,353	276,531	345,506
Vt.	314,120	315,098	330,551	332,286	332,422
Oreg.	13,294	52,465	90,923	174,768	313,767
D.C.	51,687	75,080	131,700	177,624	230,392
Del.	91,532	112,216	125,015	146,608	168,493
Nev.		6,857	42,491	62,266	45,761
Rest	72,927	184,497	311,030	606,819	1,621,118

§ 3. Combinations — Three Figures.

Declare the sums:

	9	1	8	8	8	7	5	7	4	6	1	1	1	1	3	7	4	9	2	2
94.	9,	7,	8,	7,	3,	3,	4,	8,	4,	7,	1,	1,	5,	7,	8,	7,	9,	9,	6,	8.
	2	9	2	2	3	1	3	9	5	8	8	9	9	9	9	7	9	9	9	9
	3	4	5	8	6	3	2	1	1	2	5	6	8	8	8	5	9	1	1	1
95.	7,	7,	5,	9,	6,	4,	2,	4,	6,	5,	7,	7,	8,	8,	8,	5,	9,	4,	8,	9.
	8	9	9	9	9	4	6	8	8	9	8	9	8	9	5	3	3	9	9	9
	4	5	9	8	3	2	2	2	2	3	3	3	4	4	6	3	5	4	6	5
96.	8,	6,	6,	8,	5,	5,	2,	3,	5,	5,	3,	9,	4,	8,	8,	4,	9,	4,	9,	8.
	8	8	5	3	9	8	9	7	5	3	4	9	4	9	9	6	9	7	9	9
	3	7	4	6	7	4	7	2	7	1	1	1	3	4	4	4	3	4	4	5
97.	3,	8,	5,	8,	8,	4,	7,	4,	9,	6,	5,	4,	7,	5,	6,	6,	5,	7,	5,	7.
	6	7	5	8	8	6	9	9	9	6	6	5	9	9	9	8	8	8	8	7
	3	1	6	6	1	6	2	3	1	1	1	4	1	1	3	4	5	2	2	1
98.	6,	1,	6,	7,	1,	6,	3,	8,	1,	8,	2,	4,	1,	1,	3,	6,	6,	2,	2,	3.
	6	1	7	7	2	8	3	9	3	8	2	9	4	5	3	6	6	3	4	3
	3	1	3	5	2	1	2	1	1	3	1	4	1	5	2	5	1	6	1	4
99.	7,	3,	6,	6,	3,	2,	2,	2,	2,	6,	2,	6,	4,	5,	2,	5,	3,	6,	1,	7.
	7	4	9	7	4	4	2	6	3	8	5	7	4	7	5	8	5	6	7	7
	5	1	4	3	2	2	2	2	1	1	1	2	1	1	2	2	2	1	1	2
100.	5,	1,	5,	5,	6,	5,	4,	4,	2,	4,	3,	6,	2,	7,	4,	7,	2,	6,	3,	3.
	6	6	7	7	6	6	6	4	8	7	6	8	8	8	7	7	8	9	8	5
	2	2	1	1	7	2	3	3	2	1	4	2	3	3	3	1	3	1	1	4
101.	4,	6,	2,	3,	7,	5,	3,	3,	3,	5,	5,	3,	3,	4,	6,	4,	6,	5,	4,	5,	5.
	8	7	9	9	1	7	9	7	9	7	7	8	8	7	9	7	5	9	8	6
	7	4	6	4	5	9	7	8	7	3	3	4	9	8	5	9	6	3	4	8
102.	7,	8,	6,	3,	2,	9,	6,	9,	4,	8,	4,	7,	2,	3,	4,	7,	6,	4,	5,	5.
	8	9	7	1	8	4	3	1	2	7	9	8	1	4	9	2	6	3	4	7
	1	6	8	9	2	5	8	5	2	3	2	7	9	6	3	7	8	7	9	9
103.	4,	7,	2,	8,	9,	4,	7,	4,	1,	4,	4,	4,	5,	4,	3,	1,	9,	4,	6,	5.
	5	8	1	7	7	3	6	3	9	3	2	7	8	9	8	9	2	1	3	4

Ex. **94.** 20, 17, 18, 17, 14, 11, 12, etc. Look upon $\frac{9}{9}$ as 18; then $\frac{9}{9}$ $\frac{9}{2}$ appears $\frac{18}{2}$; say 20. Speak no words except the sums. Allow 14 seconds for each example.

§ 4. Combinations — Four Figures.

Read the sums:

104.

9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
9'	5'	6'	2'	5'	7'	9'	5'	2'	8'	8'	6'	8'	8'	7'	6'	4'	7'	8'	4'
9	4	6	2	5	6	2	1	1	7	9	2	2	3	1	3	2	4	5	4

105.

9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
1'	5'	9'	2'	7'	6'	4'	3'	7'	6'	4'	7'	8'	6'	8'	5'	9'	8'	7'	9'
1	5	7	1	7	4	4	3	5	2	1	1	3	1	6	2	3	7	2	4

106.

1	2	2	3	3	4	5	3	4	9	8	7	6	9	8	7	6	9	8	7
1	1	2	2	3	3	5	1	1	2	3	4	5	3	4	5	6	4	5	6
9'	9'	8'	5'	8'	8'	6'	8'	7'	9'	4'	2'	4'	6'	6'	7'	5'	3'	8'	5'
9	8	8	5	8	7	6	3	5	1	1	2	3	4	1	2	2	1	1	3

107.

9	8	9	8	9	9	9	8	7	6	5	8	7	6	5	7	6	5	4	6
6	7	7	8	8	9	1	2	3	4	5	1	2	3	4	1	2	3	4	1
7'	4'	3'	4'	9'	8'	1'	6'	7'	8'	9'	8'	7'	6'	2'	6'	6'	7'	8'	8'
3	2	2	4	1	1	1	6	6	5	4	5	7	5	1	5	1	4	1	8

108.

1	2	3	3	6	1	5	9	4	8	3	7	2	6	2	1	9	7	5	3
8	9	6	5	8	2	6	1	5	9	4	8	3	7	4	3	1	9	7	5
5'	6'	7'	8'	9'	3'	7'	2'	6'	1'	5'	9'	4'	8'	6'	5'	3'	1'	9'	7'
3	6	9	7	5	4	8	3	7	2	6	1	5	9	8	7	5	3	1	9

109.

3	8	4	6	6	8	8	7	8	9	7	9	5	6	8	3	4	9	3	9
6	4	8	2	5	2	5	7	8	1	2	4	4	7	5	3	4	9	8	6
2'	1'	2'	8'	4'	7'	2'	7'	8'	8'	8'	7'	8'	2'	9'	6'	8'	2'	7'	2'
5	7	6	4	7	3	5	7	8	3	3	6	3	1	2	6	8	1	5	1

110.

7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
6	6	5	5	6	6	5	4	3	2	1	6	6	5	5	4	3	2	1	8
1'	2'	3'	4'	5'	6'	7'	8'	9'	5'	4'	3'	2'	1'	9'	5'	6'	7'	8'	9'
9	8	7	6	5	4	3	2	1	9	8	7	6	5	4	3	2	1	9	8

Ex. **104.** 36, 27, 30, 22, 28, 31, 29, etc. Look upon $\frac{9}{9}$ as 18; then $\frac{9}{9}$ appears as $\frac{18}{18}$; say 36. Speak no word except the sums. Allow 18 seconds for each example.

§ 5. Addends.

Add:

111. 18, 17, 11, 16, 15, 14, 13, 12, 19, 20, 17, 19, 18, 17, 19.
112. 19, 19, 18, 18, 17, 17, 16, 16, 15, 15, 14, 14, 13, 13, 12.
113. 21, 22, 23, 16, 15, 18, 11, 10, 19, 24, 16, 18, 12, 14, 15.
114. 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25.
115. 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40.
116. 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25.
117. 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10.
118. 40, 11, 39, 12, 38, 13, 37, 14, 36, 15, 35, 16, 34, 17, 33.

Ex. **111.** 35, 46, 62, 77, 91, etc.

A glance determines whether the sum of the units is *more than* 9 *or less than* 10. If more than 9, we increase the sum of the tens by 1; if less than 10, we take sum of the tens.

Ex. **111.** 18 + 17; the sum of the units is more than 10; we increase the sum of the tens by 1 and say 35; 35 + 11; the sum of the units is less than 9; we take the sum of the tens and say 46, etc.

Find the sum:

119. 78+94.
120. 86+75.
121. 73+68.
122. 74+92.
123. 83+75.
124. 43+58.
125. 63+18.
126. 94+87.
127. 63+25.

Ex. **119.** 172.

Beginning with 1 count to about 200:

128. By 19.
129. By 18.
130. By 17.
131. By 16.
132. By 15.
133. By 14.
134. By 13.
135. By 12.
136. By 11.

Ex. **128.** 1, 20, 39, 58, 77, 96, etc.

Add:

137.
2 4 6 9 4 7 7 7 3 1
7' 7' 5' 5' 2' 2' 7' 6' 5' 9.

138.
3 9 4 4 9 6 9 7 2 6
2' 8' 2' 7' 6' 3' 9' 2' 5' 3.

139.
5 2 5 4 7 8 5 5 6 7
9' 6' 1' 9' 1' 7' 5' 5' 6' 3.

140.
1 3 7 7 3 9 2 6 1 1
2' 1' 4' 7' 3' 7' 8' 6' 9' 9.

141.
7 5 4 2 4 6 8 2 7 3
1' 5' 4' 6' 4' 9' 4' 8' 5' 8.

142.
1 1 1 1 2 8 9 9 8 6
9' 8' 5' 9' 4' 7' 8' 4' 2' 3.

143.
4 6 5 3 7 4 5 6 8 9
1' 2' 4' 5' 4' 3' 5' 7' 8' 6.

144.
2 7 2 2 1 1 2 4 6 8
6' 2' 8' 4' 1' 7' 3' 5' 7' 9.

145.
6 3 9 1 4 6 2 1 3 5
7, 8, 7, 9, 8, 3, 1, 1, 4, 8.
7 5 4 7 2 5 5 8 6 3

146.
1 9 2 4 5 3 8 1 1 1
5, 6, 7, 7, 4, 2, 1, 8, 2, 1.
9 8 2 7 4 9 2 8 1 6

147.
3 4 4 6 9 3 7 1 1 1
5, 8, 4, 1, 6, 1, 9, 5, 9, 5.
1 3 4 5 2 2 1 1 1 6

148.
2 4 1 7 2 3 2 1 5 7
8, 6, 2, 4, 3, 6, 8, 9, 2, 2.
4 4 2 8 9 2 8 6 4 9

149.
2 5 4 8 3 2 4 5 5 4
6, 2, 6, 7, 5, 1, 2, 6, 7, 9.
4 5 4 8 8 9 7 2 9 4

150.
5 7 2 5 6 4 4 1 2 7
2, 8, 7, 4, 4, 5, 1, 7, 1, 2.
1 7 6 6 9 1 4 8 9 5

151.
2 6 9 4 9 2 3 6 4 7
1 4 1 7 9 7 5 9 5 2
9' 5' 5' 1' 9' 6' 2' 5' 5' 8.
3 5 1 9 2 6 1 7 9 3

152.
5 8 9 2 9 5 3 9 4 1
9 7 8 6 3 6 5 8 8 2
7' 3' 4' 4' 7' 1' 6' 2' 7' 5.
4 3 9 9 9 3 1 2 5 7

153.
3 1 3 2 3 2 2 1 3 3
2 1 2 2 2 7 8 8 6 8
4' 3' 3' 8' 6' 9' 5' 8' 2' 2.
1 9 8 4 8 6 5 9 5 7

154.
4 1 7 5 3 5 7 2 8 2
8 1 6 6 7 5 5 5 1 4
6' 6' 7' 2' 4' 6' 9' 9' 1' 7.
9 8 3 3 7 6 7 1 1 5

Ex. **137.** 9, 20, 31, 45, 51, 60, 74, etc, Look at $\frac{2}{7}$, say 9; at $\frac{4}{7}$, 20; at $\frac{6}{5}$, 31; at $\frac{9}{5}$, 45; etc.

Ex. **154.** 27, 43, 66, 82, etc. Look at $\frac{4}{8}$ $\frac{1}{6}$ $\frac{7}{9}$, say 27; at $\frac{1}{6}$, 43; at $\frac{6}{7}$, 66; etc.
$\frac{}{}$ $\frac{8}{}$ $\frac{3}{}$

By reading several figures at a glance and combining as in this section, the columns on page 13 may be added in 5 seconds each.

§ 6. Problems.

Declare the answer to each as quickly as possible without reading the problem aloud and before explaining.

If required to explain, avoid repetitions and unnecessary words.

155. A paid 16¢ for a book, 15¢ for a slate, and 5¢ for a pencil; how much did he pay in all? Explain.

Ans. 36¢. He paid in all the sum of 16¢, 15¢, and 5¢, or 36¢.

156. Jane bought some apples for 10¢, some peaches for 18¢, some plums for 20¢, and an orange for 8¢; how much did she pay for all?

157. John has 28 marbles in one bag, 16 in another, 14 in another, 35 in another, and 19 in another; how many has he in all?

158. There are 9 birds in one flock and 27 in another; how many are there in both? Explain.

159. One day I walked 5 miles and the next day 14 miles; how far did I walk in both days?

160. A man has horses in 3 pastures: in the first 9, in the second 11, and in the third 13; how many has he in all? Explain.

161. A baker sold 36 loaves of bread on Monday, 30 on Tuesday, 27 on Wednesday, 34 on Thursday, 25 on Friday, and 40 on Saturday; how many loaves did he sell during the week?

162. A man has 5 baskets of eggs: in the first basket there are 24 eggs, in the second 36, in the third 18, in the fourth 12, and in the fifth 16; how many has he in the five baskets?

163. 38 years, 29 years, 10 years, 19 years, 23 years, and 14 years are how many years in all?

164. Walter saw three flocks of prairie chickens; the first contained 28 chickens, the second 19, and the third 33; how many chickens did he see?

165. I bought on account: a cabbage for 10 cents, a dozen eggs for 24 cents, a peck of apples for 15 cents, a bushel of potatoes for 65 cents, and a quart of beans for 20 cents; how much do I owe the merchant for these?

166. How many books are there in the Bible if the Old Testament contains 39 and the New Testament 27 books?

167. In a factory there are 15 men, 12 women, 17 girls, and 21 boys at work; how many persons are employed in the factory?

168. Alfred earned 44 cents one day, 50 cents the next day, and found 35 cents; how many cents did he then have?

169. Clay studied 25 minutes one afternoon, 55 minutes the next afternoon, and 12 minutes the next afternoon; how many minutes did he study in all?

170. A farmer raised 36 bushels of wheat, 18 bushels of oats, 27 bushels of rye, and 19 bushels of corn; how many bushels of grain did he raise?

171. A lady canned, during the summer, 12 quarts of peaches, 9 quarts of cherries, 26 quarts of strawberries, 17 quarts of blackberries, 13 quarts of raspberries, and 19 quarts of gooseberries; how many quarts of fruit has she for winter use?

172. Find the number of days in the first six months of the year when January has 31 days, February 29 days, March 31 days, April 30 days, May 31 days, and June 30 days.

173. Find the sum of 62, 15, 9, 32, 27, 18, 8, 6, and 4.

174. A carpenter used 7 bunches of lath for the kitchen, 16 bunches for the dining-room, 12 bunches for the parlor, and 11 bunches for a bed-room; how many bunches did he use for the four rooms?

175. Bought berries for 17 cents, cherries for 19 cents, and apples for 13 cents; what was the cost of all?

176. A lady bought a dress for $18, a muff for $16, a shawl for $17, and other articles for $19; what was the whole bill?

177. A merchant sold 18 barrels of flour one week, 16 the next week, 12 the next, 13 the next, and 14 the next; how many barrels did he sell during the five weeks?

178. Mary had 56 oranges, and Susan had 19 more than Mary; how many had Susan?

179. A man is 48 years old, and his wife is 36 years old: what is the sum of their ages?

180. James had 59 cents and found 48 cents; how many cents had he then?

181. Mary gave 58 cents to her brother and 96 cents to her sister; how many cents did she give away?

182. A merchant sold rice for $198, sugar for $18, oil for $17, candy for $13, molasses for $16, and salt for $12; how much did he receive for all?

183. A girl made 15 red pin-wheels, 16 blue ones, and 17 blue and white. How many pin-wheels did she have?

184. It is 38 miles from A to B, 19 miles from B to C, 17 miles from C to D, 18 miles from D to E; how many miles does a man travel who goes from A to E, passing through B, C, and D?

SUBTRACTION.

Subtraction is indicated by the sign $-$.

The number to be subtracted is the *subtrahend.*

The number from which to subtract is the *minuend.*

The result is the *difference* or *remainder.*

The difference between any two numbers may be found by counting.

Subtraction is a process shorter than counting for finding the difference between numbers.

ILLUSTRATION.

$8 - 3 = 5$

read

8 minus 3 equals 5.

8, *minuend.*

3, *subtrahend.*

5, *difference* or *remainder.*

To find the difference between 8 *and* 3 *by counting.*

Counting to 8 and making a mark at each count, ////////; counting to 3 and crossing a mark at each count, XXX/////; counting what is left, we have 5.

Read the remainders as rapidly as possible:

1.	10	10	10	10	10	10	10	10	10	11	11	11	11	11	11
	9	5	3	7	4	1	8	2	6	9	6	2	4	8	3
2.	11	11	12	12	12	12	12	12	12	13	13	13	13	13	13
	5	7	9	5	8	4	6	3	7	9	5	7	4	8	6
3.	14	14	14	14	14	15	15	15	15	16	16	16	17	17	18
	9	7	5	8	6	9	7	8	6	9	7	8	9	8	9

Ex. 1. 1, 5, 7, 3, 6, 9, etc. Do not say 9 from 10 leaves 1.

What must be added to:

4. 9 to make 18?
5. 9, 8 to make 17?
6. 7, 9, 8 to make 16?
7. 8, 9, 7, 6 to make 15?
8. 7, 9, 5, 6, 8 to make 14?
9. 5, 8, 7, 9, 6, 4 to make 13?
10. 6, 7, 5, 4, 8, 3, 9 to make 12?
11. 5, 8, 7, 4, 3, 6, 2, 9 to make 11?
12. 9, 2, 3, 5, 4, 7, 6, 1, 8 to make 10?

Ex. **8.** 7, 5, 9, 8, 6.

Beginning with 100 count backwards:

13. By 9.
14. By 8.
15. By 7.
16. By 6.
17. By 5.
18. By 4.
19. By 3.
20. By 2.
21. By 1.

Ex. **13.** 100, 91, 82, etc.

Begin at the *right* and read the remainder:

22.	23.	24.	25.
98736	5897638	12345678	98768975
86224	2634521	1343527	82345723
26.	**27.**	**28.**	**29.**
85672	5463267	76924711	64723108
43461	3252145	56412300	23412106

Ex. **22.** Say 2, 1, 5, 2, 1. The habit of saying "4 from 6 leaves 2; 2 from 3 leaves 1; 2 from 7 leaves 5," etc., should be broken up. While the four words, "4 from 6 leaves," are being formed, no progress can be made in subtracting. There is no reason why the student should not call off the figures of the remainder as rapidly as he can talk.

Begin at the right and read the remainder:

30.	31.	32.	33.
36854	3000205	43000005	568120001
29876	1864783	17652436	497203854

34.	35.	36.
503784325	6234567890	3050702003
282958298	3929802958	1234567898

37.	38.
286540302567200	806304205102030
192830605088739	507080610503028

39.	40.
736904521300671	192470030060091
497000457369712	141398765432189

41.	42.
862300100439610	572000000700123
765489012786937	324986574309165

43.	44.
432176090135200	678230004000500
168349827563425	437654321234567

Ex. **30.** Say 8, 7, 9, 6, and *no other words.* Practice will enable the student to read the figures of the remainder *almost* as rapidly as he can talk.

45. To make 100, what must be added to 56? 48? 32? 74? 83? 76? 25? 73? 44? 86? 92? 38? 53? 27? 49? 33? 18? 58? 67? 77?

46. To make 1000, what must be added to 72? 102? 148? 156? 63? 179? 185? 196? 144? 156? 175? 183? 122? 104? 157? 163? 177? 192?

47. To make 1000, what must be added to 676? 687? 575? 762? 349? 534? 296? 105? 428? 777? 388? 499?

48. To make 1000, what must be added to 375? 804? 783? 926? 439? 604? 593? 355? 707? 599?

49. To make 10000, what must be added to 3608? 5732? 4963? 6078? 7095? 2801? 5678? 4209?

50. From 1000000, subtract 886097, 407864, 360835, 479632, 582769, 380803, 760967, 320457, 978654.

51. From 100000000, subtract 23456789, 92037405, 50640720, 40009265, 70904055, 66090207.

52.	**53.**	**54.**
1000000000	1000000000	1000000000
372840625	102030458	289076430
55.	**56.**	**57.**
1000000000	1000000000	1000000000
203572763	123456789	246813579
58.	**59.**	**60.**
1000000000	1000000000	1000000000
764031246	837964012	543212345

Ex. **48.** 625, 196, 217, etc. In these examples, it is best to begin at the left and call out what must be added to each figure of the subtrahend except the last to make 9, but what must be added to the *last* figure to make 10. *The student should read results as fast as he can talk.*

61. From 276 take 189.
62. From 364 take 278.
63. From 467 take 389.
64. From 123 take 74.
65. From 106 take 98.
66. From 207 take 199.
67. From 111 take 46.
68. From 203 take 145.
69. From 209 take 167.
70. From 245 take 169.
71. From 456 take 389.
72. From 506 take 489.
73. From 287 take 198.
74. From 802 take 746.
75. Take 123 from 210.
76. Take 299 from 323.
77. Take 145 from 223.
78. Take 345 from 421.
79. Take 258 from 324.
80. Take 456 from 531.
81. Take 489 from 503.
82. Take 286 from 345.

Ex. **61**. 87. To make 200, 11 must be added to 189; $76 + 11 = 87$. Do not say, "9 from 16 leaves 7; 9 from 17 leaves 8."

83. To make 922, what must be added to 648? 396? 479? 553? 764? 875? 283? 697? 785? 892? 189? 527? 819? 634? 311? 439? 510? 609?

84. To make 816, what must be added to 378? 496? 785? 396? 519? 439? 382? 786? 758? 729? 715? 638? 525? 444? 775? 314? 678? 248?

85. To make 725, what must be added to 648? 639? 675? 686? 695? 683? 681? 649? 663? 671? 535? 598? 419? 307? 212? 199? 25? 63?

86. To make 513, what must be added to 416? 438? 269? 183? 68? 75? 233? 175? 254? 285? 54? 19? 153? 240? 369? 452? 387? 299?

Ex. **83**. 274, 526, 443, etc. Consider what must be added to each to make 900; then add 22; *e.g.* to make 900, 252 must be added to 648; $252 + 22 = 274$; to make 900, 504 must be added to 396; $504 + 22 = 526$, etc.

§ 7. Problems.

Declare the answers to each as quickly as possible without reading the problem aloud and before explaining.

If required to explain, avoid repetitions and unnecessary words.

87. From a box containing 37 marbles 29 were lost; how many remained? Explain.

Ans. 8 marbles. There remained the difference between 37 marbles and 29 marbles, or 8 marbles.

88. Henry had 75¢ and gave 45¢ for a knife; how many had he left?

89. A man had $145 in the bank and drew out $89; how many dollars had he left in the bank? Explain.

90. A merchant bought 124 barrels of apples, and sold 98 barrels; how many had he left?

91. A farmer had 115 sheep and after selling some he had 86 left; how many did he sell? Explain.

92. In an orchard there are 100 peach trees and 57 plum trees; how many more peach trees than plums trees are there?

93. A receives a salary of $125 per month, and after paying his necessary expenses he has $57 left; what are his expenses? Explain.

94. B borrowed $105 and paid $79 of the debt; how much did he still owe?

95. The sum of two numbers is 122, one of the numbers is 39; what is the other? Explain.

96. A pays $130 for a horse and a saddle. He pays $13 for the saddle; how much does he pay for the horse?

97. From a bin containing 112 bushels of corn, 76 bushels were sold; how many bushels remained?

98. From a school of 109 pupils, 18 were absent; how many were present? Explain.

99. Ray's father gave him 75¢, his mother gave him 45¢, his sister gave him 20¢, and his brother gave him 15¢. He spent 92¢; how much had he left? Explain.

100. The sum of three numbers is 145; the first is 24, the second is 48; what is the third?

101. A merchant had on hand 36 pounds of butter; he bought 95 pounds more and then sold 57 pounds; how much had he left?

102. What is the difference between $69+82$ and $73+49$?

103. What is the difference between $132-29$ and $110-61$?

104. What is the difference between $101+48+24$ and $111-19$?

105. What number subtracted from $98+23$ will leave 73?

106. Ethel had 45¢; she spent 29¢, after which she earned 57¢; how many cents had she then?

107. A farmer raised 150 bushels of wheat; he sold at one time 35 bushels, at another time 49 bushels and kept the remainder; how many bushels did he keep?

108. A earns \$100 per month, and pays \$15 for board and \$46 for other expenses; how much does he save each month?

109. From a piece of carpet containing 76 yards, two pieces were cut and 29 yards remained; the first piece cut off contained 13 yards; how many yards did the second piece contain? Explain.

110. Subtract 87 from 104, add 45 to the remainder, and declare the result.

111. On Monday a gentleman deposited in a bank \$53, on Tuesday he deposited \$85, on Wednesday he drew out \$99; how much did he leave in the bank?

112. A owes me 76¢; I owe him 91¢; how may we settle the account? Explain.

113. From the sum of 65 and 88 subtract the sum of 37 and 49.

114. James bought 100 oranges and sold 68 of them; how many had he left?

115. John is 77 years old, and Joseph is 38; John is how many years older than Joseph?

116. In a school of 66 pupils 29 are present; how many are absent?

117. Of 1000 men 128 were sick; how many were well?

118. From a herd of 256 cattle 189 were sold; how many remained?

119. A man sold 48 cows, then bought 19, and then had 65; how many had he at first?

120. Prove that your answer to the 119th is correct.

121. A farmer exchanged eggs costing 35¢, butter 19¢, potatoes $1.05, for cloth costing 36¢, sugar 50¢, starch 12¢; how much was due him?

122. A man bought a horse for $57, received for his use $19, and paid for his keeping $12; he sold him for $65; how much did he gain?

123. Paid $37 for sugar, $29 for molasses; how much did both cost? How much more did the sugar cost than the molasses?

124. John bought 35 apples at one store and 48 at another; he sold 29 of them; how many remained?

125. I bought a horse for $65; for how much must I sell him to gain $38?

126. A farmer sold a cow for $38, which was $19 more than she cost him; how much did he pay for her?

MULTIPLICATION.

Multiplication is indicated by the sign $\times$.

The number to be multiplied is the *multiplicand.*

The number by which to multiply is the *multiplier.*

The result is the *product.*

Any number of times a given number may be found by adding.

Multiplication is a process shorter than adding for finding the sum of equal addends.

ILLUSTRATION.

$6 \times 4 = 24$

read

6 multiplied by 4 equals 24

or

4 times 6 equals 24.

6, *multiplicand.*

4, *multiplier.*

24, *product.*

To multiply 6 *by* 4 *by adding.*

$6 \times 4 = 6 + 6 + 6 + 6$

or

6 taken 4 times as an addend.

$\therefore\ 6 \times 4 = 24.$

×	1	2	3	4	5	6	7	8	9	10	11	12
12	12	24	36	48	60	72	84	96	108	120	132	144
11	11	22	33	44	55	66	77	88	99	110	121	132
10	10	20	30	40	50	60	70	80	90	100	110	120
9	9	18	27	36	45	54	63	72	81	90	99	108
8	8	16	24	32	40	48	56	64	72	80	88	96
7	7	14	21	28	35	42	49	56	63	70	77	84
6	6	12	18	24	30	36	42	48	54	60	66	72

$13 \times 13 = 169$	$18 \times 18 = 324$	$24 \times 24 = 576$	$5 \times 5 \times 5 = 125$
$14 \times 14 = 196$	$19 \times 19 = 361$	$25 \times 25 = 625$	$6 \times 6 \times 6 = 216$
$15 \times 15 = 225$	$21 \times 21 = 441$	$2 \times 2 \times 2 = 8$	$7 \times 7 \times 7 = 343$
$16 \times 16 = 256$	$22 \times 22 = 484$	$3 \times 3 \times 3 = 27$	$8 \times 8 \times 8 = 512$
$17 \times 17 = 289$	$23 \times 23 = 529$	$4 \times 4 \times 4 = 64$	$9 \times 9 \times 9 = 729$

When a number is multiplied by itself, as 13×13, 14×14, etc., the product is called a *square*. When the square of a number is multiplied by the number, as $2 \times 2 \times 2$, $3 \times 3 \times 3$, etc., the product is called a *cube*.

Declare the products of:

1. 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, by 9.
2. 9, 6, 2, 5, 11, 4, 2, 7, 3, 8, 10, by 8.
3. 2, 10, 6, 9, 4, 12, 8, 11, 7, 5, 3, by 7.
4. 10, 4, 9, 3, 8, 12, 2, 7, 11, 6, 5, by 6.
5. 6, 8, 4, 10, 2, 9, 12, 3, 11, 7, 5, by 5.
6. 6, 9, 2, 8, 3, 7, 10, 12, 4, 11, 5, by 4.
7. 12, 4, 11, 2, 10, 6, 9, 2, 8, 5, 7, by 11.
8. 11, 2, 7, 3, 2, 4, 6, 5, 8, 12, 9, by 12.

Ex. 2. 72, 48, 16, 40, etc. Do not say, "8 times 9 are 72."

Declare the products of:

9. 13 by 1, 2, 3, 4, 5, 6, 7.
10. 14 by 1, 2, 3, 4, 5, 6, 7.
11. 15 by 1, 2, 3, 4, 5, 6.
12. 16 by 1, 2, 3, 4, 5, 6.
13. 17 by 1, 2, 3, 4, 5.
14. 18 by 1, 2, 3, 4, 5.
15. 19 by 1, 2, 3, 4.
16. 21 by 1, 2, 3, 4.
17. 22 by 1, 2, 3, 4.
18. 23 by 1, 2, 3, 4.
19. 24 by 1, 2, 3, 4.
20. 25 by 1, 2, 3, 4.

21. 3 by 33, 27, 31, 28, 26, 29, 30, 32.

22. 2 by 49, 30, 39, 31, 42, 32, 47, 48, 28, 38, 29, 46, 27, 36, 26, 41, 33, 45, 43, 37, 34, 40, 35.

23. 13×13, $2 \times 2 \times 2$, 14×14, $8 \times 8 \times 8$.

24. 14×14, $9 \times 9 \times 9$, 25×25, $4 \times 4 \times 4$, 21×21, $7 \times 7 \times 7$, 17×17, $5 \times 5 \times 5$, 19×19, $6 \times 6 \times 6$, 15×15, $3 \times 3 \times 3$, 16×16, 24×24.

Ex. 9. 13, 26, 39, 52, etc. Do not say, "13 times 1 are 13."

Give the multiplication table:

25. '13 *times*' to 13×7.
26. '14 *times*' to 14×7.
27. '15 *times*' to 15×6.
28. '16 *times*' to 16×6.
29. '17 *times*' to 17×5.
30. '18 *times*' to 18×5.
31. '19 *times*' to 19×5.
32. '21 *times*' to 21×4.
33. '22 *times*' to 22×4.
34. '23 *times*' to 23×4.
35. '24 *times*' to 24×4.
36. '25 *times*' to 25×4.

Ex. **25.** 13×1 are 13; 13×2 are 26; 13×3 are 39; etc.

Declare the results rapidly:

37. 19×5, 17×4, 16×4, 14×4, 18×5, 19×4, 16×6, 13×3, 5×16, 17×5, 5×13, 17×3, 4×13, 16×3, 19×3.

38. 19×2, 3×18, 17×2, 2×16, 15×6, 5×14, 15×5, 14×3, 13×2, 4×18, 15×3, 2×14, 13×6, 7×14, 2×18.

39. 4×15, 6×16, 7×13, $9 \times 5 \times 2$, $2 \times 3 \times 7$, 8×13, 4×14, 6×15, 6×13, $2 \times 6 \times 8$, $5 \times 6 \times 2$, 6×14, 18×4, 14×6.

40. 16×5, 15×2, 14×2, 15×4, 14×5, 14×7, $3 \times 4 \times 5$, $2 \times 4 \times 6$, $5 \times 4 \times 2$, $3 \times 5 \times 6$.

State rapidly:

41. The squares of the integers from 1 to 25.

42. The cubes of the integers from 1 to 9.

43. The square of 25, 23, 24, 21, 19, 16, 17, 15, 13, 22, 20, 18.

44. The square of 12, 14, 16, 18, 20, 22, 24, 13, 15, 17, 19, 21.

45. The cube of 9, 6, 3, 1, 4, 7, 8, 5, 2, 5, 7, 9, 4, 6, 8.

46. The cube of 4, 3, 5, 7, 9, 2, 4, 6, 8, 10, 9, 2, 8, 6, 7.

Multiply:

47.	48.	49.
2030876514	5708392943	3886546312
9	8	7
50.	**51.**	**52.**
4571263972	7360925168	2784235879
4	12	11
53.	**54.**	**55.**
1203045678	6543712345	6789098765
8	6	5
56.	**57.**	**58.**
4630902034	5762198345	6290311269
11	7	8
59.	**60.**	**61.**
3862048001	3572607983	6238496785
9	3	12
62.	**63.**	**64.**
4837254912	5789564328	7912765046
4	6	11

Ex. **47.** 36, 12, 46, 58, 68, etc. Do not say, "9 times 4 are 36; 9 times 1 are 9 and 3 are 12; 9 times 5 are 45 and 1 are 46," but declare the results only.

§ 8. Problems.

Declare the answers to each as quickly as possible without reading the problem aloud and before explaining.

If required to explain, avoid repetitions and unnecessary words.

65. At 13¢ each what will 12 baskets cost? Explain.

Ans. 156¢. Since 1 basket costs 13¢, 12 baskets will cost 12 times 13¢, or 156¢.

66. Is the following a correct explanation of Ex. 65? *Since* 1 *basket costs* 13¢, 12 *baskets will cost* 13 *times* 12¢, *or* 156¢.

No. Because the denomination of 12 is baskets and not cents.

67. At $3 per barrel what will 19 barrels of flour cost?

68. At $9 per head what will 21 sheep cost?

69. If a man travels 5 miles an hour, how far will he travel in 16 hours? Explain.

70. A train runs 29 miles an hour; how far will it run in 8 hours?

71. If a man earns $6 per week, how much does he earn in 12 weeks?

72. How much will 4 acres of land cost at $57 an acre?

73. If 34 men can do a piece of work in 11 days, how long will it take one man to do it? Explain.

74. If 3 pipes fill a cistern in 19 hours, how long will it take one pipe to fill it?

75. Henry is 12 years old, and his father is 5 times as old; how old is his father?

76. If a ship sails 9 miles an hour, how far will it sail in 9 hours?

77. A farmer bought 19 yards of cloth at 3¢ a yard, and gave in exchange 5 dozen eggs at 16¢ a dozen; how much was due him? Explain.

Ans. 23¢. If 1 yard cost 3¢, 19 yards cost 19 times 3¢, or 57¢; if 1 dozen eggs sold for 16¢, 5 dozen sold for 80¢; if the cloth cost 57¢ and the eggs brought 80¢, the farmer's due was the difference, or 23¢.

When the scholar has thoroughly mastered this form, he should be required to abbreviate the explanation. Thus, the cloth cost 57¢; the eggs brought 80¢; therefore 23¢ was due the farmer.

78. A man bought 7 yards of cloth at 16¢ a yard, and 5 yards of cloth at 12¢ a yard; what was the entire cost?

79. If the income from one cow is $16 per year, and from one sheep $2, what is the income from 6 cows and 8 sheep?

80. I paid 18¢ each for 12 chickens, and 2¢ each for 13 eggs; what was the entire cost?

81. What will be the cost of 5 pictures at 19¢ each, and 7 hooks at 3¢ each?

82. A lady bought 6 dozen buttons at 12¢ per dozen, and gave in payment one dollar; how much change should she receive?

83. A man bought 6 barrels of apples at $3 per barrel, and gave in exchange 12 sacks of flour at $2 per sack; how much was due him?

84. James bought 8 marbles for 5¢ each, 6 pencils for 10¢ each, and a book for 25¢; he gave in payment 150¢; how much change should he receive?

85. Harvey bought 9 oranges for 7¢ each, and 11 lemons for 5¢ each; he gave in exchange 9 pounds of butter at 15¢ per pound; how much was due him?

86. John traded 8 marbles worth 8¢ each, for a jack-knife worth 50¢ and some money; how much money should he receive?

87. What is the profit in buying 6 cows at $20 each, and selling at $25 each?

Ans. $30. It is best to find the profit on *one*. Thus, the profit on *one* cow is $5; on 6 cows, $30.

88. What is the profit in buying 10 shares of stock at $99 each, and selling them at $103 each?

89. What is received from the sale of 16 cows at $21 each, if $1 each is paid the agent for selling them?

90. What is received from the sale of 16 shares of stock at $101 each, if $1 each is paid the broker for selling them?

91. How much does a man gain by buying 6 cows at $21 each, paying an agent $1 each for purchasing, and selling them at $27 each, paying an agent $2 each for selling them?

92. Does a man gain or lose and how much by buying 6 shares of stock at $101 each, paying a broker $1 each for purchasing, and selling at $103 each, paying a broker $2 each for selling them?

93. When beef is 5¢ a pound, and pork 6¢ a pound, how much more will 17 pounds of beef cost than 14 pounds of pork?

94. Which costs the more, the keeping of 16 horses 9 weeks at $1 a week each, or the keeping of 12 cows 12 weeks at 50¢ a week each? How much more?

95. Two persons start from the same point and travel in opposite directions: one travels 5 miles an hour, and the other 7 miles an hour; how far apart are they at the end of 13 hours?

96. How far apart are they at the end of 13 hours if they travel in the same direction?

97. Which is cheaper and how much per dozen, to buy eggs at 25¢ a dozen or at 3¢ each?

DIVISION.

Division is indicated by the sign ÷.

The number to be divided is the *dividend.*

The number by which to divide is the *divisor.*

The result is the *quotient.*

That which remains when the division is not exact is the *remainder.*

The number of times one number is contained in another may be found by subtracting.

Division is a process shorter than subtracting for finding how many times one number is contained in another.

ILLUSTRATION.

$20 \div 7 = 2\frac{6}{7}$

read

20 divided by 7 = 2 and 6 remainder.

20, *dividend.*
7, *divisor.*
2, *quotient.*
6, *remainder.*

To divide 9 *by* 4, *by subtracting.*

9 ÷ 4 calls for the number of times that 4 may be subtracted from 9.

$9 - 4 - 4 = 1$;

that is,

$9 \div 4 = 2\frac{1}{4}$.

Declare the quotients of:

1. 144, 96, 36, 60, 72, 48, 24, 84, 108, 132, 120, divided by 12.
2. 88, 44, 77, 132, 99, 121, 66, 22, 55, 33, 110, divided by 11.
3. 72, 108, 81, 54, 63, 18, 45, 90, 27, 108, 36, divided by 9.
4. 64, 56, 96, 24, 32, 16, 72, 40, 80, 48, divided by 8. By 4.
5. 63, 14, 35, 56, 21, 70, 84, 28, 42, 35, 49, 70, divided by 7.
6. 54, 36, 18, 72, 54, 12, 24, 48, 42, 60, divided by 6. By 3.
7. 25, 40, 30, 15, 50, 35, 55, 20, 60, 45, 10, 65, divided by 5.
8. 10, 18, 6, 16, 22, 14, 20, 8, 12, 4, 24, 28, 32, divided by 2.

Ex. 1. 12, 8, 3, 5, 6, etc. Do not say, "144 ÷ 12 are 12."

Declare the quotient and remainder of:

9. 119, 111, 113, 81, 117, 86, 118, 77, 116, 90, 87, ÷12.
10. 109, 71, 106, 80, 102, 79, 105, 60, 107, 59, 86, ÷11.
11. 89, 60, 76, 83, 52, 63, 50, 73, 62, 55, 80, 25, ÷ 9.
12. 79, 70, 61, 62, 73, 71, 67, 60, 78, 68, 63, 65, ÷ 8.
13. 69, 59, 62, 54, 58, 23, 67, 57, 64, 55, 60, 53, ÷ 7.
14. 59, 50, 41, 55, 51, 27, 57, 47, 46, 53, 45, 56, ÷ 6.
15. 49, 41, 44, 38, 36, 22, 46, 39, 48, 34, 42, 37, ÷ 5.
16. 39, 30, 33, 37, 26, 22, 34, 23, 31, 25, 29, 35, ÷ 4.
17. 29, 19, 17, 23, 11, 16, 8, 28, 20, 22, 10, 13, ÷ 3.

Ex. 9. 9, 11; 9, 3; 9, 5; 6, 9; etc. Do not say, "119 ÷ 12 are 9 and 11 remaining."

Declare the quotient and remainder of:

18. 12 contained in 119, 17, 111, 14, 113, 13, 117, 18, 118, 15, 116, 16, 114, 19, 112, 28, 110, 24, 115, 20, 109, 25, 106, 26, 102, 21, 105, 23, 107, 22, 101, 27, 108, 29, 104, 34, 100, 30, 103, 38, 99, 35, 97, 37, 90, 32, 93, 33, 98, 31, 96, 36.

19. 12 contained in 91, 39, 94, 40, 95, 48, 92, 44, 89, 41, 85, 47, 80, 43, 88, 46, 84, 42, 81, 45, 86, 49, 82, 87, 53, 83, 57, 79, 52, 73, 55, 77, 51, 74, 58, 78, 50, 76, 56, 70, 54, 75, 59, 71, 64, 69, 68, 66, 62, 63, 60, 67, 65, 61, 72.

20. 11 contained in 109, 65, 106, 11, 102, 12, 105, 17, 107, 14, 101, 13, 108, 18, 104, 15, 100, 16, 103, 19, 99, 28, 97, 24, 90, 20, 93, 25, 98, 26, 96, 21, 91, 23, 94, 22, 95, 27, 92, 29, 89, 34, 85, 30, 80, 38, 88, 77, 35, 71, 37, 66, 32, 69, 33, 79, 31, 54.

21. 9 contained in 89, 10, 80, 12, 84, 14, 86, 18, 87, 16, 79, 28, 77, 20, 78, 26, 70, 22, 71, 29, 66, 30, 67, 35, 65, 32, 62, 31, 64, 39, 54, 48, 50, 41, 51, 43, 52, 42, 53, 49, 45, 57, 46, 55, 47, 58, 44, 56, 40, 59, 36, 68, 33, 60, 37, 61, 38, 63, 34, 69, 27.

Ex. 18. 9, 11; 1, 5; 9, 3; 1, 2; etc.

Read the quotient:

22. 12)119076324

23. 11)207806035

24. 9)803706256

25. 7)369246810

26. 6)121416181

27. 3)920212233

28. 4)245678900

29. 5)312760030

30. 8)405001762

31. 3)902637052

32. 7)510000900

33. 12)634310271

34. 8)507603295473286023

35. 7)123456780924572568

36. 6)102003040507623086

37. 5)102305067052342125

38. 4)816540092367813362

39. 3)276300009823145911

40. 9)923634373532146874

41. 11)382900768419582123

42. 12)823476298345151718

43. 6)102030456783961048

44. 5)345678987654321235

45. 4)953872641219382116

46. 9)876678543345210038

47. 11)398476521830057621

Ex. **22.** 9, 9, 2, 3, 0, 2, 7. Speak no words except the quotient figures. Do not say, "12 into 119, 9 times and 11 over; 12 into 110, 9 times and 2 over."

State results rapidly:

48. 7 times 14 are how many times 2? 13? 12? 5? 7?
49. 9 times 11 are how many times 6? 12? 8? 9? 6?
50. 5 times 15 are how many times 3? 16? 9? 8? 7?
51. 4 times 17 are how many times 2? 15? 8? 3? 6?
52. 2 times 33 are how many times 3? 11? 22? 2? 8?
53. 9 times 10 are how many times 5? 18? 15? 6? 3?
54. 6 times 16 are how many times 3? 32? 6? 12? 24?
55. 8 times 9 are how many times 2? 18? 4? 6? 8? 12? 24?
56. 5 times 12 are how many times 2? 3? 4? 6? 10? 15?

State results rapidly:

57. $9\times 5, +15$ are how many times 12? 15? 4? 7?
58. $14\times 6, -7$ are how many times 11? 7? 18? 6?
59. $7\times 12, -8$ are how many times 19? 13? 14? 15?
60. $6\times 8, +4$ are how many times 13? 6? 9? 7?
61. $10\times 4, +2$ are how many times 6? 8? 9? 12?
62. $8\times 9, +16$ are how many times 8? 12? 16? 9?
63. $14\times 4, +8$ are how many times 16? 8? 4? 32?
64. $18\times 3, +6$ are how many times 15? 12? 5? 10?
65. $17\times 5, +5$ are how many times 18? 10? 16? 9?

State results rapidly:

66. How many times 21 are 14×6? 7×9? 3×28?
67. How many times 24 are 6×16? 8×6? 6×12?
68. How many times 13 are 39×2? 39×3? 26×2?
69. How many times 12 are 15×4? 6×16? 54×2?
70. How many times 11 are 33×3? 22×4? 44×2?
71. How many times 9 are 12×12? 6×12? 5×18?
72. How many times 14 are 7×8? 2×49? 7×10?

Declare the results rapidly:

73. $24 \div 12$; $25 \div 5$; $26 \div 13$; $27 \div 9$; $28 \div 7$; $30 \div 15$; $32 \div 16$; $33 \div 11$; $34 \div 17$; $35 \div 5$; $36 \div 18$; $38 \div 19$.

74. $39 \div 13$; $40 \div 8$; $42 \div 14$; $44 \div 4$; $45 \div 15$; $48 \div 16$; $49 \div 7$; $50 \div 10$; $51 \div 3$; $52 \div 4$; $54 \div 3$; $55 \div 11$.

75. $56 \div 4$; $57 \div 3$; $60 \div 4$; $64 \div 4$; $65 \div 5$; $66 \div 6$; $68 \div 4$; $70 \div 7$; $72 \div 18$; $75 \div 5$; $76 \div 4$; $77 \div 11$.

76. $78 \div 6$; $80 \div 5$; $81 \div 9$; $84 \div 6$; $85 \div 5$; $88 \div 11$; $90 \div 18$; $91 \div 13$; $95 \div 19$; $96 \div 16$; $98 \div 7$; $99 \div 11$.

Declare the results rapidly:

77. $96 \div 16$, 12, 24, 3, 8, 32, 48, 4, 6; $98 \div 2$, 7, 49, 14.

78. $99 \div 11$, 33, 9, 3; $94 \div 2$, 47; $93 \div 3$, 31; $92 \div 2$, 23, 46, 4; $91 \div 7$, 13; $90 \div 9$, 5, 18, 3, 45, 30, 2, 15, 6.

79. $88 \div 11$, 2, 22, 44, 4, 8; $87 \div 3$, 29; $86 \div 2$, 43.

80. $85 \div 5$, 17; $84 \div 7$, 21, 12, 14, 4, 6, 42, 2, 3, 28; $81 \div 9$, 27, 3; $80 \div 16$, 8, 10, 5, 40, 20.

81. $78 \div 39$, 2, 13, 6, 3; $76 \div 38$, 2, 19, 4; $74 \div 37$, 2; $72 \div 18$, 12, 24, 6, 3, 4, 36, 2, 4, 8, 9.

82. $70 \div 35$, 7, 5, 14, 2; $64 \div 8$, 4, 16, 32; $63 \div 9$, 3, 7, 21.

83. $54 \div 6$, 3, 9, 27; $48 \div 8$, 4, 6, 2, 24, 3, 16.

Name sets of two numbers each, whose product is:

84. 99, 98, 96, 95, 94, 93, 92, 91, 90, 88, 87, 86.

85. 85, 84, 82, 81, 80, 78, 77, 76, 75, 74, 72, 70.

86. 69, 68, 66, 65, 64, 63, 62, 60, 58, 57, 56, 55.

87. 54, 52, 51, 50, 49, 48, 46, 45, 44, 42, 40, 39.

88. 38, 36, 35, 34, 33, 32, 30, 28, 27, 26, 25, 24.

89. 22, 21, 20, 18, 16, 15, 14, 12, 10, 9, 8, 6, 4.

Ex. **84.** 33 and 3; 9 and 11. 7 and 14; 49 and 2. 6 and 16; 8 and 12; 4 and 24; 3 and 32; 2 and 48; etc.

§ 9. Precedence of Signs.

What is the value of $6+4\times5$?

Mathematicians have agreed to use the sign '×' before the sign '+' or '−.'

If the signs are used as they occur, the answer is 50; if the sign '×' is used first, 26.

What is the value of $6-4\div2$?

Mathematicians have agreed to use the sign '÷' before the sign '+' or '−.'

If the signs are used as they occur, the answer is 1; if the sign '÷' is used first, the answer is 4.

What is the value of $24\div4\times2$?

There is no agreement as to which sign shall be used first. It is best to avoid such expressions.

If the signs are used as they occur, the answer is 12; if the sign '×' is used first, the answer is 3.

What is the value of $6-4+8$?

It makes no difference in what order the signs '+' and '−' are used.

If the signs are used as they occur, the answer is 10; if the sign '+' is used first, the answer is 10.

Find the value of:

90. $6+8\div2-3+2$.

91. $72\div6-64\div8-3$.

92. $6\times8\div12+4$.

93. $96\div16+72\div24-8$.

94. $99\div11-81\div9+25$.

95. $7+8\div4+9\times2-12\div4$.

96. $9+16\div8-18\div3+2\times5$.

97. $25+10\div5-27\div3$.

98. $8+4-7+6-9\div3$.

99. $18+15\div3+72\div24$.

100. $30+5\div5-36\div3-8$.

101. $92\div23+87\div3+49-7$.

102. $98\div7-42\div6+18\div9$.

103. $33\div3-10\div2-8\div4$.

104. $64\times2\div32+88\div8-11$.

105. $96\div32+84\div12-70\div7$.

Ex. **90.** 9. Say 6, 10, 7, 9.

§ 10. Parenthesis or Bar.

To indicate that several quantities are to be subjected to the same operation, they are written within curved lines or brackets, or under or over a straight line.

Commas are sometimes used to indicate that the signs are to be used in the order of their occurrence.

Illustration.

$(6+3)\times 5$,

or

$\overline{6+3}\times 5$,

means,

the sum is to be multiplied by 5.

read,

the expression 6 *plus* 3 *times* 5.

6, +8, ÷2, ×7

means,

to 6, *add* 8, *divide by* 2, *multiply by* 7.

Find the value of:

106. $[(9\times 8+9)\div 9+5]\div 2+8$.

107. $(6+8)\div 2+(5-3)\times 2$.

108. $(7\times 5-3\times 8)\times 8+4$.

109. $(8\times 12-18\times 4)\div(9\times 7-19\times 3)$.

Ex. **106.** 15.

Find the value of:

110. 9, ×8, +6, ÷13, ×5, ÷2, ×4, +4, ÷8, +9, ×5.

111. 90, +9, ÷11, ×2, +6, ÷8, ×7, +4, ÷5, ×8, +2, ÷7, ×8, +1, ÷7.

112. 19, ×4, +5, ÷9, ×5, +6, ÷17, ×18, +7, +3, ÷16.

113. 18, ×5, +6, ÷8, ×12, −44, +8, ÷12, ×9, +7.

114. 17, ×4, +5, +8, +6, ÷3, +2, +5, +14, ÷10, ×8, +9, +8, −1, ÷7.

Ex. **110.** 7. Say 9, 72, 78, 6, 30, 15, 60, 64, 8, 17, 85.

Find the value of:

115. 6, +7, ×5, +5, +2, ÷18, ×4, +4, ÷5, +3, ×6, +2, ÷11, ×8, +1, ÷3, ×7, +8.

116. 98, ÷7, +7, +8, +7, ÷12, ×29, +3, +12, +30, ÷12, ×11, ×3, ÷11, +12, ÷5, +1.

117. 16, +17, −18, ×6, +1, ÷13, ×14, +2, ÷10, +25, +18, +22, ÷15, ×18, +9, ÷9, −9.

118. 45, ÷15, ×3, ×5, ×2, ÷6, ×4, ÷5, ×12, ÷16, ×8, ÷9, ×5, ÷10, ×7, ×2, ÷7.

119. 2, +19, −16, +15, +18, −13, −12, +9, +8, −17, +25, −13, +18, −6, −9, −5, +4.

120. 18, +17, −19, −12, +23, +48, −16, −19, +13, −11, +12, −9, +16, −8, +17, −18, −19.

121. 13, +12, −11, +15, −14, +22, −19, −3, +8, +9, +17, +13, −6, −9, −12, −25, +18.

122. 7, ×7, +1, ÷5, ×8, +5, ÷17, ×6, +2, ÷16, ×49, ÷7, ×6, +6, ÷18, +6, ×8, +4, ÷23, +6.

123. 19, +11, +13, +2, +16, +5, +14, +18, +4, +7, +13, +10, +9, +12, +14, +16, +6, +8, +11.

124. 245−18−13−15−16−12−11−10−9−8−7−6−5−4−3−2−1−11−14−17−25−8.

125. 19, +8, −7, −6, +13, −14, +18, −17, +16, +12, −14, −15, +9, −8, +7, −6, −11.

126. 4, ×5, +1, ÷7, ×5, +2, +6, −3, ÷4, ×6, +2, ÷8, ×4, +4, ÷2, ×8, +1, ÷9, ×6, +2, ÷7.

127. 5, ×12, +3, ÷7, ×8, −5, −3, ÷8, ×6, +1, ÷7, ×6, +2, ÷11, ×4, ×4, +7, +8, −11, ÷17.

128. 6, ×7, +9, −3, ÷8, ×6, +4, ÷5, ×3, +1, ÷5, ×6, +6, ÷18, ×10, +1, ÷7, ×6, +4, ÷11.

129. 8, ×5, +2, ÷7, ×6, +4, ÷10, ×14, +6, +9, −8, ÷7, ×2, +4, ÷2, ×11, +11, ÷12.

§ 11. Principles.

Multiplying both dividend and divisor by the same number does not affect the quotient.

$24 \div 4 = 6.$
$48 \div 8 = 6.$

Dividing both dividend and divisor by the same number does not affect the quotient.

$24 \div 4 = 6.$
$12 \div 2 = 6.$

Multiplying the dividend multiplies the quotient.

$24 \div 4 = 6.$
$48 \div 4 = 12.$

Dividing the dividend divides the quotient.

$24 \div 4 = 6.$
$12 \div 4 = 3.$

Multiplying the divisor divides the quotient.

$24 \div 4 = 6.$
$24 \div 8 = 3.$

Dividing the divisor multiplies the quotient.

$24 \div 4 = 6.$
$24 \div 2 = 12.$

Division is expressed in four ways:

Eight divided by three is expressed:

By writing the dividend above and the divisor below a horizontal line.

$\frac{8}{3}$, *fractional method.*

By writing the sign '÷' between the terms.

$8 \div 3$, *common method.*

By writing the sign ':' between the terms.

8 : 3, *ratio method.*

By writing the divisor at the left, and the dividend at the right, of a curved line.

3)8, *working method.*

The first method was originally used. Later, to get both terms on the same horizontal line the dividend was written, then the line '—' with a dot over it for the dividend and a dot below for the divisor, then the divisor. Later the line was omitted.

§ 12. Problems.

Declare the answer to each as quickly as possible without reading the problem aloud and before explaining.

If required to explain, avoid repetitions and unnecessary words.

130. If 14 apples cost 28¢, what will 1 apple cost? Explain.

Ans. 2¢. If 14 apples cost 28¢, 1 apple will cost $\frac{1}{14}$ of 28¢, or 2¢.

Ans. 2¢. If 14 apples cost 28¢, 1 apple will cost as many cents as 14 is contained times in 28, or 2¢.

131. What is the cost of 1 yard of cloth when 16 yards cost 96¢?

132. A man divided $200 among 20 persons; how much did each receive?

133. If 18 yards of cloth cost $54, for how much must it be sold per yard to gain $36?

134. A farmer gave 18 barrels of flour, worth $4 a barrel, for 12 yards of cloth; how much was the cloth a yard?

135. If 24 hours equal 360 degrees, how many degrees equal 1 hour?

136. If 360 degrees equal 24 hours, how many hours equal 1 degree?

137. Eleven cows were sold for $220; what was the selling price of each?

138. Eleven shares of stock were sold for $1111; what was the selling price of each?

139. Eleven cows were sold for $231, and $1 per cow was paid to the agent for selling them; how much did the owner receive for each cow?

140. If 9 tables cost $108, what will 12 tables cost?

141. At 3¢ each, how many pears can be bought for 39¢?

Ans. 13 pears. Since 1 pear costs 3¢, 39¢ will buy as many pears as 3 is contained times in 39, or 13 pears.

142. Would this explanation be correct? "*If one pear costs* 3¢, 39¢ *will buy as many pears as* 3¢ *is contained times in* 39¢, *or* 13 *pears.*"

Yes. Because 3¢ is contained times in 39¢.

143. Would this explanation be correct? "*Since* 1 *pear costs* 3¢, 39¢ *will buy as many pears as* 3¢ *is contained times in* 39."

No. Because 3¢ is not contained times in 39.

144. Would it be right to say, "*If* 1 *pear costs* 3¢, 39¢ *will buy as many pears as* 3 *is contained times in* 39¢"?

145. At 4¢ each, how many lemons can I buy for 72¢?

146. If 1 cow costs $15, how many can be bought for $75?

147. At $19 each, how many sheep can be bought for $95?

148. A and B started together in the same direction from the same point, A at the rate of 5 miles an hour, and B at the rate of 3 miles an hour; in how many hours will A be 14 miles ahead of B?

149. Traveling as before, B has 20 miles the start; in how many hours will A overtake B?

150. A and B started at the same time from the same point in opposite directions, with the same rate as before; how far apart will they be in 10 hours?

151. After traveling 10 hours they turn around towards home. Who will reach home first? How far will A have traveled? How far will B have traveled?

152. If 16 oranges are worth 32 pears, and 3 pears are worth 6 apples, and apples are worth 2¢ each, how many oranges can be bought for 40¢?

153. If 6 quarts of berries cost 18¢, what will 12 quarts cost?

Ans. 36¢. 1 quart will cost $\frac{1}{6}$ of 18¢, or 3¢; 12 quarts will cost 12 times 3¢, or 36¢.

Ans. 36¢. 12 quarts are 2 times 6 quarts; 12 quarts will cost 2 times 18¢, or 36¢.

154. If 12 pounds of cheese cost 108¢, what will 36 pounds cost?

155. How many pounds of butter, at 15 cents per pound, must be given for 18 pounds of sugar at 5 cents a pound?

156. If 8 sheep cost $80, how much will 24 sheep cost?

157. If 5 men can do a piece of work in 20 days, in how many days can 4 men do it?

158. How many barrels of flour can be bought for $40, if 5 barrels cost $50?

159. How long will it take Paul to earn 99 cents, if he earns 18 cents in 2 weeks?

160. How many years will it take to pay a debt of $1080, if $720 are paid in 6 years?

161. How much will 24 barrels of apples cost, if 6 barrels cost $24?

162. At 24¢ for 12 apples, what will 72 apples cost?

163. At 18¢ for 3 dozen clothes-pins, how many clothes-pins can be bought for 30¢?

164. If 19 apples cost 57¢, what will 14 apples cost?

165. If 19 apples cost 57¢, how many apples can be bought for 51¢?

166. If 17 books cost $153, what will 22 such books cost?

167. At $185 for 5 cloaks, what will 7 cloaks cost?

168. If 23 cows sell for $920, at the same rate what will 30 cows bring?

FACTORING.

A number may exactly contain another; the container is a *multiple;* the contained, a *factor.*

A number may have other factors besides itself and *one;* a *composite number.*

A number may have no other factors besides itself and *one;* a *prime number.*

Several numbers may have no common factor greater than *one; numbers prime to each other.*

Each of several numbers may be prime to each of the others; *numbers severally prime.*

ILLUSTRATION.

12 contains 6, 2 times.

12, *a multiple of* 6.

6, *a factor of* 12.

12, a *composite number.* Its factors are, 1, 2, 3, 4, 6, 12.

7, a *prime number.* It has no factors except 7 and 1.

8, 12, 25, are prime to each other.

8, 9, 25, 49, are severally prime.

Name:

1. All the composite numbers from 1 to 100.
2. All the prime numbers from 1 to 100.
3. Two composite numbers prime to each other.
4. Three numbers prime to each other.
5. Three numbers severally prime.

Define:

6. A *multiple* of a number.
7. A *factor* of a number.
8. A *composite* number.
9. A *prime* number.
10. Numbers *prime to each other.*
11. Numbers *severally prime.*

A number is divisible:

By 2, when its last digit is divisible by 2.

By 5, when the number denoted by its last digit is divisible by 5.

By 4, when the number denoted by its last two digits is divisible by 4.

By 8, when the number denoted by its last three digits is divisible by 8.

By 3, when the sum of its digits is divisible by 3.

By 9, when the sum of its digits is divisible by 9.

By 11, when the difference between the sum of its digits in the odd places and the sum of its digits in the even places is divisible by 11.

By the product of any number of its factors which are severally prime to each other.

ILLUSTRATION.

3960 is divisible by 2, because 0 is divisible by 2.

3960 is divisible by 5, because 0 is divisible by 5.

3960 is divisible by 4, because 60 is divisible by 4.

3960 is divisible by 8, because 960 is divisible by 8.

3960 is divisible by 3, because 18, the sum of its digits, is divisible by 3.

3960 is divisible by 9, because 18, the sum of its digits, is divisible by 9.

3960 is divisible by 11, because 0, the difference between 9 (the sum of its digits in the odd places) and 9 (the sum of its digits in the even places) is divisible by 11.

3960 is divisible by the product of $3 \times 4 \times 5 \times 11$, or 660, because 3, 4, 5, and 11 are factors of 3960, and are severally prime.

Which of the numbers 2, 3, 4, 5, 8, 9, 11, are factors of:

12. 27720?	**15.** 48532?	**18.** 72754?
13. 3960?	**16.** 9768?	**19.** 3675?
14. 6732?	**17.** 19998?	**20.** 14175?

Ex. **12.** 2, 3, 4, 5, 8, 9, 11.

Name all the following numbers that are factors of 360360:

21. 2, 3, 4, 5, 7, 8, 9, 11, 13, 17, 19, 23.

22. 2×3, 2×4, 2×5, 2×7, 2×8, 2×9.

23. 2×11, 2×13, 3×4, 3×5, $3\times7\times2$.

24. 3×9, 3×11, 3×13, 4×5, $4\times7\times3$.

25. 4×9, 4×11, 4×13, 5×7, $5\times8\times3$.

26. 5×11, 5×13, 7×8, 7×9, $7\times11\times2$.

27. 8×9, 8×11, 8×13, 9×11, 9×13.

28. 2×9, 3×8, 4×8, 5×9, 7×13, 6×7.

29. $2\times3\times8$, $2\times4\times5$, $2\times5\times9$, 6×9.

30. $2\times7\times9$, $2\times8\times11$, $3\times4\times5\times8$.

31. $3\times4\times11$, $3\times5\times9$, $3\times7\times9$, 11×12.

32. $9\times11\times13$, $7\times11\times13$, $5\times11\times13$.

33. $3\times8\times11$, $3\times11\times13$, $4\times5\times11\times2$.

34. $4\times7\times11$, $4\times9\times11$, $5\times7\times9\times4$.

35. $2\times3\times4$, $2\times3\times5$, $2\times3\times7$, 20×7.

36. $2\times3\times9$, $2\times3\times11$, $2\times3\times13\times5$.

37. $2\times4\times7$, $2\times5\times7$, $2\times5\times8$, 12×15.

38. $2\times5\times11$, $2\times5\times13$, $2\times7\times8\times5$.

39. $2\times7\times11$, $2\times7\times13$, $2\times8\times9\times7$.

40. $2\times9\times11$, $2\times9\times13$, $2\times11\times13$.

41. $3\times4\times7$, $3\times4\times8$, $3\times4\times9\times7\times11$.

42. $3\times4\times13$, $3\times5\times7$, $3\times5\times8\times11\times2$.

43. $3\times5\times11$, $3\times5\times13$, $3\times7\times8\times9$.

44. $3\times7\times11$, $3\times7\times13$, $3\times8\times9\times2$.

45. $3\times8\times13$, $3\times9\times11$, $3\times9\times13\times5$.

46. $3\times5\times11\times13$, $4\times8\times9\times11\times13$.

47. $5\times7\times8\times13$, $5\times7\times8\times11\times9\times13$.

48. $5\times8\times9\times11$, $5\times8\times9\times13\times2\times3$.

49. $3\times5\times9\times11$, $3\times4\times7\times9\times11\times13$.

Ex. 29. $2\times5\times9$, because 2, 5, 9 are severally prime.

Of the following, some of the factors are given. Find two or three more for each, by taking the product of factors severally prime:

50. Number 360; factors 4, 9, 8.
51. Number 1155; factors 3, 7, 5, 11.
52. Number 1260; factors 12, 15, 7.
53. Number 600; factors 3, 4, 10, 5.
54. Number 210; factors 15, 14, 7, 2.
55. Number 660; factors 20, 33, 3.
56. Number 2520; factors 5, 7, 8, 9, 4, 3, 6.
57. Number 2431; factors 11, 13, 17.

Ex. **53.** 12 or 3×4, 30 or 3×10, 15 or 3×5, etc.

By inspection tell why:

58. 1224 is divisible by 72.
59. 3465 is divisible by 55.
60. 2394 is divisible by 63.
61. 10208 is divisible by 88.
62. 10197 is divisible by 99.
63. 2034 is divisible by 18.
64. 1463 is divisible by 77.
65. 3144 is divisible by 24.
66. 980 is divisible by 35.
67. 1728 is divisible by 72.

Ex. **58.** 1224 is divisible by 8 and by 9; hence by 8×9 because 8, 9 are severally prime.

By inspection determine a common factor of:

68. 36, 48, 72.
69. 77, 88, 121, 22.
70. 96, 56.
71. 360, 144, 9872.
72. 235, 25.
73. 300, 250, 400.
74. 3260, 84, 96.
75. 395, 95, 625.
76. 88, 84, 90.
77. 378, 117, 234.

§ 13. Multiplication and Division.

A number expressed by its factors will be multiplied by a number, if any *one* of its factors is multiplied by that number.

A number expressed by its factors will be divided by a number, if any *one* of its factors is divided by that number.

Illustration.

$$\frac{4\times6\times8}{2}=4\times6\times16 \qquad \frac{4\times6\times8}{2}=4\times12\times8 \qquad \frac{4\times6\times8}{2}=8\times6\times8$$

$$2)\underline{4\times6\times8}=2\times6\times8 \qquad 2)\underline{4\times6\times8}=4\times3\times8 \qquad 2)\underline{4\times6\times8}=4\times6\times4$$

Multiply:

78. $2\times3\times4$ by 6.
79. $7\times5\times8$ by 7.
80. $9\times7\times3$ by 4.
81. $8\times2\times5$ by 3.
82. $9\times6\times7$ by 2.
83. $2\times3\times4$ by 5×6.
84. $3\times4\times5$ by 2×3.
85. $6\times5\times7$ by 4×5.
86. $8\times3\times9$ by 2×6.
87. $7\times9\times8$ by 2×3.

Ex. **78.** $2\times3\times24$, $2\times18\times4$, or $12\times3\times4$.

Ex. **83.** $10\times18\times4$, or $2\times15\times24$, or $10\times3\times24$, etc.

Divide:

88. $9\times18\times6$ by 3.
89. $12\times10\times8$ by 5.
90. $17\times3\times6$ by 17.
91. $14\times18\times12$ by 6.
92. $18\times24\times36$ by 12.
93. $54\times64\times18$ by 18×8.
94. $72\times96\times48$ by 24×8.
95. $81\times72\times44$ by 9×11.
96. $76\times24\times34$ by 19×17.
97. $48\times36\times24$ by 16×12.

Ex. **88.** $3\times18\times6$, or $9\times6\times6$, or $9\times18\times2$.

Ex. **93.** $3\times8\times18$, or $54\times8\times1$.

Divide:

98. $85 \times 6 \times 7$ by 17.
99. $95 \times 8 \times 3$ by 19.
100. $95 \times 8 \times 3$ by 24.
101. $80 \times 9 \times 7 \times 11$ by 88.
102. $96 \times 12 \times 7 \times 11$ by 112.
103. $72 \times 8 \times 7$ by 168.
104. $64 \times 7 \times 9$ by 56.
105. $70 \times 2 \times 6$ by 14.
106. $90 \times 3 \times 7$ by 54.
107. $78 \times 3 \times 7$ by 39.
108. $75 \times 4 \times 8$ by 50.
109. $52 \times 8 \times 7$ by 91.
110. $9 \times 12 \times 14$ by 36; 42; 108.
111. $56 \times 8 \times 9$ by 64; 72; 168.
112. $26 \times 8 \times 12$ by 24; 32; 104.
113. $75 \times 84 \times 16$ by 25; 16; 175.
114. $96 \times 35 \times 17$ by 48; 14; 672.
115. $84 \times 20 \times 16$ by 28; 40; 480.
116. $19 \times 18 \times 14$ by 28; 36; 126.
117. $75 \times 42 \times 28$ by 56; 42; 100.
118. $30 \times 70 \times 20$ by 14; 24; 210.
119. $17 \times 19 \times 18$ by 34; 38; 153.
120. $20 \times 21 \times 22$ by 28; 56; 154.
121. $23 \times 24 \times 25$ by 92; 30; 115.
122. $26 \times 27 \times 28$ by 13; 63; 117.
123. $29 \times 30 \times 31$ by 58; 62; 186.
124. $32 \times 33 \times 34$ by 88; 44; 136.
125. $44 \times 45 \times 46$ by 55; 22; 460.
126. $47 \times 48 \times 49$ by 47; 21; 112.
127. $62 \times 63 \times 64$ by 93; 16; 288.
128. $65 \times 66 \times 67$ by 26; 67; 143.
129. $68 \times 69 \times 70$ by 69; 23; 115.

Ex. **98.** $5 \times 6 \times 7$. $85 \div 17 = 5$.

Ex. **101.** $10 \times 9 \times 7$. Factors 88 are 8 and 11; $80 \div 8 = 10$; $11 \div 11 = 1$.

Ex. **127.** $2 \times 21 \times 64$. Factors 93 are 31 and 3; $62 \div 31 = 2$; $63 \div 3 = 21$.

§ 14. Greatest Common Divisor.

The G. C. D. of two or more numbers is the product of all the common factors which may be used as successive divisors until the quotients are prime to each other.

Illustration.

12	72	144	108
3	6	12	9
	2	4	3

$12 \times 3 = 36 =$ G. C. D.

The G. C. D. of two numbers is the G. C. D. of the smaller and of the remainder found by dividing the greater by the smaller.

32)70(2 G. C. D. 32, 70 is 2.
64 G. C. D. 32, 6 is 2.
6

One of the numbers may be divided by a number prime to one of the others without affecting the G. C. D.

25)75 G. C. D. 75, 36 is 3.
3 G. C. D. 3, 36 is 3.

By the second principle, find the G. C. D. of:

130. 64, 96.	**134.** 35, 75.	**138.** 46, 69.
131. 56, 84.	**135.** 44, 90.	**139.** 40, 60.
132. 72, 108.	**136.** 27, 84.	**140.** 32, 48.
133. 24, 76.	**137.** 36, 75.	**141.** 38, 57.

Ex. **130.** 32. The G. C. D. of 64 and 96 is the G. C. D. of 64 and 32 (the remainder), or 32.

By the third principle, find the G. C. D. of:

142. 75, 96.	**146.** 36, 44.	**150.** 35, 91.
143. 98, 72.	**147.** 22, 36.	**151.** 72, 56.
144. 46, 68.	**148.** 77, 91.	**152.** 33, 75.
145. 51, 72.	**149.** 80, 64.	**153.** 45, 95.

Ex. **142.** 3. $75 \div 25 = 3$. G. C. D. of 75 and 96 is the G. C. D. of 3 and 96, or 3.

§ 15. Least Common Multiple.

To find the L. C. M. of two numbers, divide one of them by their G. C. D., and multiply the quotient by the other.

Illustration.
L. C. M. 10 and 12 is 60.
$12 \div 2 = 6$.
$10 \times 6 = 60$.

To find the L. C. M. of more than two numbers, find the L. C. M. of two of them, then of the result and a third, and so on.

L. C. M. 10, 12, 15 is 60.
L. C. M. 10, 12 is 60.
L. C. M. 60, 15 is 60.

If one of the numbers exactly contains another, the smaller may be neglected.

L. C. M. 12, 24 is 24.
12 may be neglected.

By the first principle, find the L. C. M. of:

154. 12, 14.	**165.** 60, 72.	**176.** 78, 52.
155. 15, 12.	**166.** 25, 30.	**177.** 68, 85.
156. 16, 20.	**167.** 30, 40.	**178.** 95, 57.
157. 24, 32.	**168.** 24, 27.	**179.** 90, 72.
158. 40, 50.	**169.** 96, 84.	**180.** 80, 96.
159. 60, 80.	**170.** 49, 63.	**181.** 42, 63.
160. 30, 26.	**171.** 72, 96.	**182.** 84, 63.
161. 14, 21.	**172.** 48, 52.	**183.** 56, 42.
162. 18, 20.	**173.** 75, 80.	**184.** 76, 72.
163. 32, 36.	**174.** 60, 72.	**185.** 50, 75.
164. 36, 40.	**175.** 35, 42.	**186.** 65, 26.

Ex. **154.** 84. The G. C. D. of 12, 14 is 2; $14 \div 2 = 7$; $12 \times 7 = 84$.

COMMON FRACTIONS.

§ 16. First Conception — An Expression of Division.

Division may be expressed by writing the dividend above, and the divisor below, a line. Such an expression is a *common fraction;* the dividend is the *numerator;* the divisor, the *denominator.*

The numerator, or the denominator, or both, may contain fractions; such an expression is a *complex fraction.*

We sometimes speak of a fraction of a fraction; a *compound fraction.*

An integer plus a fraction is a *mixed number.* The plus sign is usually omitted.

Analyze $\frac{3}{4}$. $\frac{\frac{2}{3}}{\frac{3}{4}}$.

Define by first conception:

1. A common fraction.
2. A complex fraction.
3. A compound fraction.
4. A mixed number.
5. The numerator.
6. The denominator.

Illustration.

$\frac{4}{5}$, *common fraction.*

It means $4 \div 5$.

4, *numerator.*

5, *denominator.*

read, $4 \div 5$.

$\frac{\frac{2}{3}}{4}$, $\frac{4}{\frac{5}{6}}$, $\frac{\frac{7}{8}}{\frac{9}{14}}$, *complex fractions.*

$\frac{2}{3}$ of $\frac{5}{6}$, *a compound fraction.*

$5\frac{2}{3}$, *a mixed number.*

$\frac{3}{4}$; 3 is the numerator; 4, the denominator; it means $3 \div 4$.

$\frac{2}{3}$ is the numerator; $\frac{3}{4}$, the denominator; it means $\frac{2}{3} \div \frac{3}{4}$.

§ 17. Second Conception — One or More of the Equal Parts of a Unit.

A unit may be divided into two or more equal parts, and one or more of those parts may be taken.

The number denoting into how many parts the unit is divided is written below a horizontal line, and is the *denominator*.

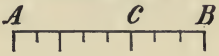
The number showing how many parts are taken is written above the line, and is the *numerator*.

The whole expression is a *common fraction*.

According to this conception, is $\frac{8}{5}$ a fraction? No. It is called an *improper fraction*, *i.e.* not properly a fraction.

According to this conception, is $\frac{\frac{2}{3}}{\frac{5}{4}}$ a fraction?

Illustration.



AB is divided into 8 equal parts; AC contains 5 of them; $AC = 5$ eighths of AB; expressed, $AC = \frac{5}{8}$ of AB.

$\frac{5}{8}$, *common fraction.*
5, *numerator.*
8, *denominator.*
read, 5 *eighths.*

It means that a unit is divided into 8 equal parts and 5 of those parts are taken.

$\frac{1}{2}$, read *one half*; $\frac{3}{4}$, read 3 *quarters*, or 3 *fourths.*

It is impossible to divide a unit into 5 equal parts and then take 8 of them.

No. It is impossible to divide a unit into $\frac{3}{4}$ equal parts.

Define by second conception:

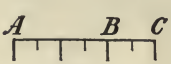
7. A common fraction.
8. The denominator.
9. The numerator.
10. An improper fraction.

§ 18. Change of Form — To Higher Terms.

Multiplying both numerator and denominator by the same number does not change the value of a fraction.

This is to prepare fractions for addition and subtraction.

Illustration.



$AB = \frac{2}{3}$, or $\frac{4}{6}$, of AC.

$\frac{2}{3} = \frac{4}{6}$

(*multiplying both terms by* 2).

Change:

11. $\frac{3}{4}$ to 16ths.	**16.** $\frac{5}{9}$ to 36ths.	**21.** $\frac{9}{17}$ to 51sts.
12. $\frac{5}{6}$ to 12ths.	**17.** $\frac{7}{12}$ to 24ths.	**22.** $\frac{11}{14}$ to 56ths.
13. $\frac{7}{8}$ to 40ths.	**18.** $\frac{5}{13}$ to 39ths.	**23.** $\frac{13}{15}$ to 60ths.
14. $\frac{9}{16}$ to 48ths.	**19.** $\frac{7}{11}$ to 77ths.	**24.** $\frac{19}{21}$ to 84ths.
15. $\frac{5}{7}$ to 35ths.	**20.** $\frac{5}{14}$ to 28ths.	**25.** $\frac{13}{15}$ to 90ths.

Ex. **11.** $\frac{12}{16}$. To make the denominator 16, we must multiply it by 4; multiplying both terms by 4, $\frac{3}{4} = \frac{12}{16}$.

Reduce to equivalent fractions having their least common denominator:

26. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.	**31.** $\frac{1}{5}, \frac{1}{7}, \frac{1}{30}$.	**36.** $\frac{5}{14}, \frac{3}{56}, \frac{4}{7}$.
27. $\frac{5}{7}, \frac{3}{14}, \frac{8}{21}$.	**32.** $\frac{1}{8}, \frac{5}{12}, \frac{7}{24}$.	**37.** $\frac{4}{33}, \frac{5}{66}, \frac{7}{11}$.
28. $\frac{5}{9}, \frac{11}{18}, \frac{7}{36}$.	**33.** $\frac{8}{9}, \frac{7}{36}, \frac{5}{72}$.	**38.** $\frac{4}{7}, \frac{3}{28}, \frac{5}{4}$.
29. $\frac{6}{13}, \frac{5}{39}, \frac{3}{52}$.	**34.** $\frac{4}{25}, \frac{7}{50}, \frac{11}{100}$.	**39.** $\frac{9}{16}, \frac{5}{48}, \frac{7}{8}$.
30. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$.	**35.** $\frac{3}{7}, \frac{8}{77}, \frac{5}{11}$.	**40.** $\frac{4}{15}, \frac{5}{3}, \frac{7}{60}$.

Ex. **26.** $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$. Multiplying both terms of $\frac{2}{3}$ by 4; of $\frac{3}{4}$, by 3; of $\frac{5}{6}$, by 2.

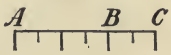
§ 19. Change of Form — To Lower Terms.

Dividing both numerator and denominator by the same number does not change the value of a fraction.

This is to reduce fractions to their simplest forms.

Which fraction is the more readily comprehended, $\frac{38}{57}$, or $\frac{2}{3}$? Why?

Illustration.



$AB = \frac{4}{6}$, or $\frac{2}{3}$, of AC.

$\frac{4}{6} = \frac{2}{3}$

(*dividing both terms by* 2).

$\frac{2}{3}$, because the terms are smaller.

Reduce to lowest terms:

41. $\frac{12}{18}, \frac{9}{27}, \frac{30}{60}, \frac{5}{25}, \frac{6}{12}$.

42. $\frac{8}{16}, \frac{10}{20}, \frac{16}{32}, \frac{11}{33}, \frac{18}{36}$.

43. $\frac{19}{38}, \frac{11}{44}, \frac{16}{48}, \frac{12}{48}, \frac{25}{50}$.

44. $\frac{14}{52}, \frac{18}{54}, \frac{29}{58}, \frac{17}{68}, \frac{18}{72}$.

45. $\frac{11}{22}, \frac{26}{39}, \frac{52}{65}, \frac{65}{78}, \frac{13}{91}$.

46. $\frac{14}{28}, \frac{42}{70}, \frac{84}{98}, \frac{28}{42}, \frac{42}{56}$.

47. $\frac{30}{60}, \frac{45}{60}, \frac{75}{90}, \frac{30}{45}, \frac{60}{75}$.

48. $\frac{12}{84}, \frac{28}{84}, \frac{56}{84}, \frac{70}{84}, \frac{42}{84}$.

49. $\frac{12}{96}, \frac{32}{96}, \frac{64}{96}, \frac{48}{96}, \frac{72}{96}$.

50. $\frac{48}{72}, \frac{80}{96}, \frac{32}{64}, \frac{16}{80}, \frac{5}{45}$.

51. $\frac{39}{78}, \frac{70}{98}, \frac{15}{30}, \frac{14}{84}, \frac{24}{96}$.

52. $\frac{13}{39}, \frac{46}{92}, \frac{84}{96}, \frac{132}{144}, \frac{77}{121}$.

53. $\frac{112}{336}, \frac{144}{1728}, \frac{125}{625}, \frac{625}{1250}$.

54. $\frac{19}{133}, \frac{28}{98}, \frac{56}{98}, \frac{63}{112}, \frac{77}{84}, \frac{35}{63}$.

55. $\frac{169}{130}, \frac{196}{140}, \frac{225}{150}, \frac{256}{160}, \frac{289}{170}$.

56. $\frac{46}{138}, \frac{30}{150}, \frac{25}{200}, \frac{19}{152}, \frac{23}{253}$.

57. $\frac{39}{351}, \frac{76}{532}, \frac{55}{385}, \frac{105}{525}, \frac{92}{368}$.

58. $\frac{74}{444}, \frac{75}{180}, \frac{60}{320}, \frac{95}{335}, \frac{54}{297}$.

59. $\frac{324}{360}, \frac{361}{570}, \frac{400}{800}, \frac{441}{840}, \frac{484}{880}$.

60. $\frac{529}{690}, \frac{576}{720}, \frac{625}{750}, \frac{8}{40}, \frac{27}{30}$.

61. $\frac{64}{80}, \frac{125}{500}, \frac{216}{360}, \frac{343}{420}, \frac{512}{800}$.

62. $\frac{729}{900}, \frac{24}{30}, \frac{36}{42}, \frac{48}{72}, \frac{60}{66}, \frac{77}{91}$.

63. $\frac{72}{78}, \frac{84}{90}, \frac{96}{102}, \frac{56}{63}, \frac{77}{84}, \frac{85}{102}$.

64. $\frac{9}{27}, \frac{16}{64}, \frac{25}{125}, \frac{36}{216}, \frac{49}{343}, \frac{6}{8}$.

65. $\frac{64}{512}, \frac{81}{729}, \frac{425}{850}, \frac{465}{930}, \frac{125}{375}$.

66. $\frac{223}{669}, \frac{777}{630}, \frac{805}{161}, \frac{245}{735}, \frac{825}{1650}$.

Ex. **41.** $\frac{2}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{5}, \frac{1}{2}$. Divide both terms of $\frac{12}{18}$ by 6; of $\frac{9}{27}$, by 9; etc.

§ 20. Change of Form — To a Whole or Mixed Number.

A fraction is an expression of division. It means that the numerator is to be divided by the denominator.

A mixed number is an integer plus a fraction. If the integer is reduced to an equivalent fraction having the denominator of the fraction, the two parts may be united.

To reduce a mixed number to an improper fraction is a case in addition of fractions.

Illustration.

$\frac{8}{3} = 2\frac{2}{3}$.

$\frac{6}{2} = 3$.

$2\frac{2}{3} = 2 + \frac{2}{3}$.

$2 = \frac{6}{3}$.

$\frac{6}{3} + \frac{2}{3} = \frac{8}{3}$.

therefore

$2\frac{2}{3} = \frac{8}{3}$.

Reduce to a whole or mixed number:

67. $\frac{96}{7}$, $\frac{84}{9}$.
68. $\frac{39}{14}$, $\frac{56}{13}$.
69. $\frac{95}{19}$, $\frac{84}{13}$.
70. $\frac{95}{18}$, $\frac{37}{18}$.
71. $\frac{45}{13}$, $\frac{52}{13}$.
72. $\frac{64}{14}$, $\frac{86}{17}$.
73. $\frac{90}{17}$, $\frac{45}{11}$.
74. $\frac{33}{12}$, $\frac{59}{19}$.
75. $\frac{55}{13}$, $\frac{68}{15}$.
76. $\frac{75}{16}$, $\frac{85}{13}$.
77. $\frac{94}{17}$, $\frac{87}{13}$.
78. $\frac{99}{15}$, $\frac{96}{15}$.

Ex. **67.** $13\frac{5}{7}$. Performing the indicated division, $96 \div 7 = 13\frac{5}{7}$.

Reduce to an improper fraction:

79. 5, $4\frac{5}{17}$.
80. $7\frac{1}{2}$, $8\frac{3}{4}$.
81. $9\frac{5}{8}$, $6\frac{2}{3}$.
82. $8\frac{1}{4}$, $3\frac{2}{9}$.
83. $5\frac{11}{13}$, $6\frac{9}{11}$.
84. $3\frac{11}{16}$, $4\frac{5}{17}$.
85. $5\frac{18}{19}$, $4\frac{7}{11}$.
86. $5\frac{11}{12}$, $6\frac{12}{13}$.
87. $7\frac{13}{14}$.
88. $6\frac{14}{15}$.
89. $6\frac{15}{16}$.
90. $5\frac{16}{17}$.

Ex. **79.** $\frac{5}{1}$, $\frac{73}{17}$. $4 = \frac{68}{17}$; $\frac{68}{17} + \frac{5}{17} = \frac{73}{17}$. This example is often explained: since there are $\frac{17}{17}$ in 1, in 4, there are 4 times $\frac{17}{17}$, or $\frac{68}{17}$; $\frac{68}{17} + \frac{5}{17} = \frac{73}{17}$.

§ 21. Addition and Subtraction.

Before fractions can be added or subtracted, they must be reduced to equivalent fractions having a common denominator.

The least common denominator should always be found.

Illustration.

$\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}$.

$\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$.

Find the value of:

91. $\frac{1}{2}+\frac{1}{3}$.
92. $\frac{3}{4}+\frac{2}{3}$.
93. $\frac{6}{7}+\frac{5}{8}$.
94. $\frac{3}{8}+\frac{7}{9}$.
95. $\frac{2}{3}+\frac{3}{5}$.
96. $\frac{3}{4}+\frac{1}{8}$.
97. $\frac{4}{7}+\frac{1}{2}$.
98. $\frac{5}{6}+\frac{7}{9}$.
99. $\frac{11}{13}+\frac{5}{6}$.
100. $\frac{11}{14}+\frac{5}{7}$.
101. $3\frac{1}{2}+2\frac{1}{3}$.
102. $2\frac{1}{8}+3\frac{2}{3}$.
103. $3\frac{1}{4}+4\frac{1}{8}$.
104. $5\frac{1}{5}+6\frac{1}{7}$.
105. $5\frac{1}{6}+7\frac{2}{3}$.
106. $8\frac{3}{5}+7\frac{5}{6}$.
107. $9\frac{3}{4}+8\frac{1}{3}$.
108. $12\frac{5}{7}+8\frac{3}{4}$.
109. $10\frac{5}{8}+9\frac{3}{5}$.
110. $18\frac{3}{7}+3\frac{2}{3}$.
111. $25\frac{5}{8}+6\frac{1}{9}$.

Ex. **91.** $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Find the value of:

112. $\frac{1}{2}-\frac{1}{3}$.
113. $\frac{3}{4}-\frac{9}{16}$.
114. $\frac{5}{6}-\frac{5}{12}$.
115. $\frac{4}{7}-\frac{3}{14}$.
116. $\frac{2}{3}-\frac{4}{9}$.
117. $\frac{3}{5}-\frac{2}{15}$.
118. $\frac{3}{4}-\frac{2}{3}$.
119. $5\frac{3}{4}-2\frac{7}{8}$.
120. $4\frac{2}{5}-1\frac{5}{6}$.
121. $3\frac{1}{3}-2\frac{1}{4}$.
122. $6\frac{1}{8}-4\frac{1}{2}$.
123. $5\frac{1}{4}-3\frac{2}{3}$.
124. $2-1\frac{11}{19}$.
125. $3\frac{1}{4}-\frac{1}{3}$.
126. $3\frac{1}{3}-2\frac{1}{2}$.
127. $7\frac{1}{6}-2\frac{1}{3}$.
128. $8\frac{3}{4}-7\frac{2}{3}$.
129. $7\frac{2}{3}-6\frac{3}{4}$.
130. $7\frac{3}{8}-5\frac{2}{5}$.
131. $8\frac{1}{4}-6\frac{3}{4}$.
132. $9\frac{2}{3}-4\frac{7}{9}$.

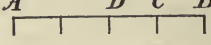
Ex. **125.** $3\frac{1}{4}-\frac{1}{3}=3\frac{3}{12}-\frac{4}{12}=2\frac{15}{12}-\frac{4}{12}=2\frac{11}{12}$.

§ 22. Multiplication — Universal Case.

Multiply the numerators for a new numerator and the denominators for a new denominator, *canceling when possible.*

Mixed numbers should be reduced to improper fractions.

Illustration.



$\frac{3}{4}$ of $AB = AC$.

$\frac{2}{3}$ of $AC = AD = \frac{1}{2}$ of AB.

$\therefore$ $\frac{2}{3}$ of $\frac{3}{4}$ of $AB = \frac{1}{2}$ of AB.

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}.$$

Find the value of:

133. $\frac{3}{14} \times \frac{16}{1}$; $\frac{5}{12} \times \frac{8}{1}$; $\frac{6}{25} \times \frac{5}{1}$.

134. $\frac{2}{3} \times 6$; $8 \times \frac{3}{4}$; $9 \times \frac{2}{3}$.

135. $\frac{3}{4} \times \frac{6}{12}$; $7 \times \frac{8}{9}$; $9 \times \frac{7}{18}$.

136. $\frac{7}{8} \times 16$; $18 \times \frac{5}{9}$; $72 \times \frac{3}{4}$.

137. $84 \times \frac{7}{12}$; $\frac{5}{6} \times 96$; $\frac{7}{8} \times 48$.

138. $96 \times \frac{3}{16}$; $\frac{11}{12} \times 84$; $\frac{9}{16} \times 64$.

139. $\frac{2}{3}$ of 18; $\frac{5}{7} \times 91$; $\frac{3}{5} \times 65$.

140. $\frac{4}{5}$ of 25; $\frac{2}{3}$ of 24; $\frac{3}{4}$ of 16.

141. $\frac{5}{7}$ of 21; $\frac{9}{11}$ of 44; $\frac{7}{13}$ of 65.

142. $\frac{2}{5}$ of 20; $\frac{9}{13}$ of 91; $\frac{8}{11}$ of 55.

143. $\frac{2}{3} \times \frac{3}{4}$; $\frac{5}{6} \times \frac{12}{25}$; $\frac{4}{9} \times \frac{9}{16}$.

144. $\frac{5}{6}$ of $\frac{6}{25}$; $\frac{3}{4}$ of $\frac{7}{9}$; $\frac{5}{12}$ of 16.

145. $\frac{9}{16}$ of $\frac{64}{63}$; $\frac{17}{54}$ of $\frac{18}{51}$; $\frac{16}{21}$ of 84.

146. $3\frac{1}{2} \times 6\frac{2}{3}$; $4\frac{3}{4} \times \frac{4}{57}$; $5\frac{1}{2} \times 12$.

147. $2\frac{2}{3} \times 3\frac{1}{8}$; $4\frac{1}{5} \times \frac{11}{21}$; $6\frac{2}{3} \times \frac{11}{20}$.

148. $5\frac{1}{6} \times \frac{11}{31}$; $2\frac{1}{2} \times 2\frac{1}{3}$; $3\frac{1}{4} \times 2\frac{1}{4}$.

149. $1\frac{1}{2} \times 1\frac{1}{3}$; $1\frac{2}{3} \times 1\frac{1}{2}$; $2\frac{2}{3} \times 2\frac{1}{2}$.

150. $\frac{3}{4}$ of $16 + \frac{2}{3}$ of $9 + 1\frac{1}{2} \times 8$.

151. $\frac{1}{2}$ of $\frac{3}{4} + \frac{1}{3}$ of $\frac{3}{4}$.

152. $\frac{2}{3}$ of $\frac{5}{6} - \frac{1}{2}$ of $\frac{1}{3}$.

153. $\frac{7}{8} \times \frac{16}{21} - \frac{3}{4}$ of $\frac{2}{3}$.

154. $3\frac{1}{2} \times \frac{2}{7} - \frac{5}{6} \times 1\frac{2}{25}$.

Ex. **133.** $\frac{24}{7}$. Divide both 16 and 14 by 2. Speak no words except the result.

Ex. **152.** $\frac{7}{18}$; $\frac{2}{3}$ of $\frac{5}{6} = \frac{10}{18}$; $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$; $\frac{10}{18} - \frac{1}{6} = \frac{7}{18}$.

§ 23. Division — Universal Case.

Divide the numerators for a new numerator and the denominators for a new denominator, *changing to equivalent fractions with their least common denominator, if necessary.*

Mixed numbers should be reduced to improper fractions.

Illustration.

$\frac{8}{15} \div \frac{2}{5} = \frac{4}{3}$.

$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$.

$3 \div 2\frac{1}{2} = \frac{6}{2} \div \frac{5}{2} = \frac{6}{5}$.

Find the value of:

155. $6 \div \frac{2}{3}$.

156. $18 \div \frac{3}{4}$.

157. $27 \div \frac{3}{5}$.

158. $84 \div \frac{7}{12}$.

159. $63 \div \frac{7}{9}$.

160. $48 \div \frac{3}{16}$.

161. $54 \div \frac{6}{13}$.

162. $77 \div \frac{7}{11}$.

163. $20 \div \frac{4}{5}$.

164. $32 \div \frac{4}{15}$.

165. $16 \div \frac{8}{15}$.

166. $25 \div \frac{5}{7}$.

167. $49 \div \frac{7}{8}$.

168. $\frac{1}{2} \div \frac{2}{3}$.

169. $\frac{3}{4} \div \frac{7}{8}$.

170. $\frac{7}{8} \div \frac{3}{4}$.

171. $\frac{3}{4} \div \frac{8}{9}$.

172. $\frac{3}{4} \div \frac{9}{8}$.

173. $\frac{5}{12} \div \frac{10}{3}$.

174. $\frac{7}{8} \div \frac{14}{11}$.

175. $2\frac{1}{2} \div 3$.

176. $2\frac{2}{3} \div 4$.

177. $3 \div 2\frac{1}{2}$.

178. $4 \div \frac{2}{3}$.

179. $1\frac{1}{2} \div \frac{1}{2}$.

180. $5 \div 3\frac{1}{3}$.

181. $5 \div 3\frac{1}{3}$.

182. $2\frac{1}{6} \div 1\frac{1}{2}$.

183. $2\frac{1}{6} \div 6\frac{1}{2}$.

184. $7\frac{3}{5} \div 8\frac{1}{5}$.

185. $7\frac{1}{5} \div 8\frac{3}{4}$.

186. $9\frac{1}{8} \div 10\frac{3}{4}$.

187. $6\frac{2}{3} \div 8\frac{1}{3}$.

188. $9\frac{5}{7} \div 6\frac{3}{7}$.

189. $4\frac{2}{3} \div 6\frac{1}{6}$.

190. $8\frac{2}{5} \div 6\frac{1}{5}$.

191. $9\frac{3}{5} \div 3\frac{1}{5}$.

192. $7\frac{4}{7} \div 2\frac{1}{7}$.

193. $8\frac{3}{5} \div 4\frac{1}{5}$.

Ex. 155. 9. $6 \div \frac{2}{3} = \frac{18}{3} \div \frac{2}{3} = 9$. If the dividend is an integer, it is easier to divide the numerator and multiply the quotient by the denominator. Thus, $6 \div \frac{2}{3}$; $6 \div 2 = 3$; $3 \times 3 = 9$.

Ex. 186. $\frac{73}{86}$. $9\frac{1}{8} \div 10\frac{3}{4} = \frac{73}{8} \div \frac{43}{4} = \frac{73}{8} \div \frac{86}{8} = \frac{73}{86}$.

Ex. 188. $\frac{68}{45}$. $9\frac{5}{7} \div 6\frac{3}{7} = \frac{68}{7} \div \frac{45}{7} = \frac{68}{45}$.

§ 24. Problems.

194. At $\frac{2}{3}¢$ each, what will 18 apples cost?

195. If 1 yard costs $18¢$, what will $\frac{2}{3}$ of a yard cost?

Ans. $12¢$. If 1 yard costs $18¢$, $\frac{2}{3}$ of a yard will cost $\frac{2}{3}$ times $18¢$, or $12¢$.

196. If 1 yard costs $18¢$, what will $5\frac{1}{2}$ yards cost?

Ans. $99¢$. If 1 yard costs $18¢$, $5\frac{1}{2}$ yards will cost $5\frac{1}{2}$ times $18¢$, or $99¢$.

Ans. $12¢$. If 1 apple costs $\frac{2}{3}¢$, 18 apples will cost 18 times $\frac{2}{3}¢$, or $12¢$.

Ans. $12¢$. If 1 yard costs $18¢$, $\frac{1}{3}$ of a yard will cost $\frac{1}{3}$ of $18¢$, or $6¢$; $\frac{2}{3}$ of a yard will cost 2 times $6¢$, or $12¢$.

Ans. $99¢$. If 1 yard costs $18¢$, 5 yards will cost $90¢$; $\frac{1}{2}$ of a yard will cost $9¢$;

$$90¢ + 9¢ = 99¢.$$

The right-hand explanation of the last two examples is objectionable because it explains the *process* of multiplication. The explanation should simply point out the operation to be employed.

197. At $\frac{3}{4}¢$ each, what will 16 oranges cost?

198. If 1 yard of cloth costs $25¢$, what will $\frac{3}{5}$ of a yard cost?

199. At $6¢$ a yard, what will $6\frac{2}{3}$ yards of cloth cost? What will $13\frac{1}{3}$ yards cost?

200. At $6\frac{2}{3}¢$ a yard, what will 6 yards of cloth cost?

201. If a man earns $\frac{3}{4}$ of a dollar per day, how much will he earn in 20 days?

202. If 1 quart pears costs $14¢$, what will $\frac{3}{7}$ of a quart cost?

203. If 1 apple costs $\frac{2}{3}¢$, what will $\frac{3}{4}$ of an apple cost?

204. If 1 orange costs $2\frac{1}{3}¢$, what will 12 oranges cost?

205. At $\frac{3}{4}¢$ each, what will 20 pears cost?

206. At $6\frac{2}{3}¢$ a yard, what will $7\frac{1}{2}$ yards of braid cost?

207. If 1 shawl costs $\$12\frac{1}{3}$, what will 15 shawls cost?

208. At $5\frac{1}{2}¢$ each, what will 16 cocoanuts cost?

209. If a boy reads $5\frac{1}{4}$ pages of a book an hour, how much will he read in $2\frac{3}{4}$ hours?

210. At $\frac{3}{4}¢$ each, how many apples can be bought for $18¢$?

Ans. 24 apples. If 1 apple costs $\frac{3}{4}¢$, $18¢$ will buy as many apples as $\frac{3}{4}$ is contained times in 18, or 24 apples.

Ans. 24 apples. If 1 apple costs $\frac{3}{4}¢$, 1 cent will buy $\frac{4}{3}$ apples; $18¢$ will buy 18 times $\frac{4}{3}$ apples, or 24 apples.

211. If $\frac{3}{4}$ of a yard of cloth costs $18¢$, what will 1 yard cost?

Ans. $24¢$. If $\frac{3}{4}$ of a yard costs $18¢$, 1 yard will cost $\frac{4}{3}$ of $18¢$, or $24¢$.

Ans. $24¢$. If $\frac{3}{4}$ of a yard costs $18¢$, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of $18¢$, or $6¢$; $\frac{4}{4}$, or one yard, will cost 4 times $6¢$, or $24¢$.

If 5 yards cost 15¢, what will 1 yard cost?

If 5 yards cost $15¢$, 1 yard will cost $\frac{1}{5}$ of $15¢$, or $3¢$. $\frac{1}{5}$ is obtained by inverting 5; in the same way we may invert $\frac{3}{4}$ in the example above.

212. At $\frac{2}{3}¢$ each, how many apples can be bought for $20¢$?

213. If $\frac{2}{3}$ of an apple costs $2¢$, what will 1 apple cost?

214. At $\frac{2}{3}¢$ each, how many apples can be bought for $18¢$?

215. If $\frac{2}{9}$ of an apple costs $4¢$, what does 1 apple cost?

216. If 1 apple costs $\frac{2}{3}¢$, how many apples can be bought for $\frac{3}{4}¢$?

217. If $\frac{2}{3}$ of an apple costs $\frac{3}{4}¢$, what does 1 apple cost?

218. If $\frac{5}{6}$ of a melon cost $20¢$, what was the cost of the whole melon?

219. If $\frac{3}{5}$ of a house cost $1200, what did the whole house cost?

220. If $\frac{3}{4}$ of a ship is worth $15,000, what is the value of the whole ship?

221. If $\frac{7}{8}$ of a farm cost $4200, what did the whole farm cost?

222. If $\frac{3}{4}$ of a store is valued at $6300, at what is the whole store valued?

223. If $\frac{6}{11}$ of a garden cost $120, what did the whole cost?

224. 12 is $\frac{2}{3}$ of what number?

Ans. 18. If 12 is $\frac{2}{3}$ of some number, that number is $12 \div \frac{2}{3}$, or 18.

Ans. 18. If 12 is $\frac{2}{3}$ of some number, $\frac{1}{3}$ of the number is $\frac{1}{2}$ of 12, or 6; $\frac{3}{3}$, or the whole number, is 3 times 6, or 18.

225. $\frac{3}{4}$ of 20 is $\frac{5}{6}$ of what number? $\frac{3}{7}$ of what?

226. $\frac{3}{5}$ of 40 is $\frac{2}{3}$ of what number? $\frac{6}{11}$ of what?

227. $\frac{7}{8}$ of 24 is $\frac{7}{9}$ of what number? $\frac{3}{4}$ of what?

228. $\frac{7}{13}$ of 52 is $\frac{4}{5}$ of what number? $\frac{14}{15}$ of what?

229. $\frac{8}{19}$ of 76 is $\frac{8}{9}$ of what number? $\frac{16}{17}$ of what?

230. $\frac{5}{17}$ of 85 is $\frac{25}{36}$ of what number? $\frac{5}{8}$ of what?

231. $\frac{3}{16}$ of 48 is $\frac{3}{7}$ of what number? $\frac{9}{16}$ of what?

232. $\frac{5}{9}$ of 45 is $\frac{5}{6}$ of what number? $\frac{25}{3}$ of what?

233. $\frac{8}{21}$ of 84 is $\frac{16}{17}$ of what number? $\frac{2}{9}$ of what?

234. $\frac{3}{7}$ of 56 is $\frac{8}{9}$ of what number? $\frac{2}{7}$ of what?

235. $\frac{5}{12}$ of 84 is $\frac{7}{8}$ of what number? $\frac{5}{9}$ of what?

236. $\frac{9}{16}$ of 80 is $\frac{9}{13}$ of what number? $\frac{15}{16}$ of what?

237. $\frac{4}{5}$ of 50 is $\frac{4}{7}$ of what number? $\frac{8}{9}$ of what?

238. $\frac{2}{3}$ of the scholars in a school, or 30, are girls; how many are boys?

239. A boy lost $\frac{1}{3}$ of his kite-string, and gave away $\frac{1}{5}$ of the remainder; he then had 400 feet; how long was the string at first?

240. Joseph is 12 years old; $\frac{2}{3}$ of Joseph's age is $\frac{2}{5}$ of John's age; how old is John?

241. The head of a fish is 15 inches long; $\frac{3}{5}$ of the length of the head is $\frac{3}{17}$ of the length of the rest of the body; what is the length of the fish?

242. In a school, $\frac{2}{3}$ of the students study algebra; $\frac{1}{4}$ of the remainder, geometry; and the rest, or 12, trigonometry; how many scholars are there?

243. At 3 for 5¢, how many apples can be bought for 20¢?

Ans. 12 apples. If 3 apples cost 5¢, 1 apple will cost $\frac{5}{3}$¢; as many apples can be bought for 20¢ as $\frac{5}{3}$ is contained times in 20, or 12 apples.

Ans. 12 apples. Since 20¢ are 4 times 5¢, 20¢ will buy 4 times 3 apples, or 12 apples.

244. If 6 quarts of berries cost 12¢, what will 17 quarts cost?

Ans. 34¢. If 6 quarts cost 12¢, 1 quart will cost $\frac{1}{6}$ of 12¢, or 2¢; 17 quarts will cost 17 times 2¢, or 34¢.

Ans. 34¢. Since 17 quarts are $\frac{17}{6}$ times 6 quarts, 17 quarts will cost $\frac{17}{6}$ times 12¢, or 34¢.

245. If 27 cans of tomatoes cost $2.70, what will 9 cans cost?

246. At 4 for 5¢, how many apples can be bought for 20¢?

247. At 2 for 3¢, how many apples can be bought for 30¢?

248. At 3 for 2¢, how many apples can be bought for 30¢?

249. If 4 quarts of berries cost $12\frac{1}{2}$¢, what will 6 quarts cost?

250. If 6 quarts of berries cost 12¢, what will 18 quarts cost?

251. If $\frac{5}{6}$ of a pound of prunes costs 10¢, what will $\frac{3}{4}$ of a pound cost?

252. If $\frac{2}{3}$ of an apple costs $\frac{3}{4}$¢, what will $\frac{5}{6}$ of an apple cost?

253. If 5 cans of tomatoes cost 60¢, what will 13 cans cost?

254. At 35¢ a dozen, what will 3 oranges cost?

255. At 20¢ a dozen, what will 9 eggs cost?

256. At 5¢ a score, what will 60 clothes-pins cost?

257. At $36 a dozen, what will eight pairs of shoes cost?

258. If 16 men can earn $32, how much can 75 men earn?

259. If $\frac{3}{4}$ of a ship's cargo is worth $2400, what is $\frac{2}{3}$ of the cargo worth?

260. If 15 sheets of paper cost 10 cents, what will 24 sheets, or one quire, cost?

261. If A can do a piece of work in 2 days, and B can do the same work in 3 days, how long will it take them working together?

Ans. $1\frac{1}{5}$ days. In 1 day, A can do $\frac{1}{2}$ of it; B can do $\frac{1}{3}$ of it; they can both do the sum of $\frac{1}{2}$ and $\frac{1}{3}$, or $\frac{5}{6}$ of it. If they can do $\frac{5}{6}$ in 1 day, it will take as many days to do $\frac{6}{6}$, or the whole, as $\frac{5}{6}$ is contained times in $\frac{6}{6}$, or $1\frac{1}{5}$ days.

262. If A and B together can do a piece of work in 5 days, and A alone in 8 days, in how many days can B alone do the work?

Ans. $13\frac{1}{3}$ days. In 1 day both can do $\frac{1}{5}$ of it; A can do $\frac{1}{8}$, and B $\frac{1}{5}-\frac{1}{8}$, or $\frac{3}{40}$ of it; it will take B as many days to do the whole as $\frac{3}{40}$ is contained times in $\frac{40}{40}$, or $13\frac{1}{3}$ days.

263. One pipe will fill a cistern in 4 hours; a second pipe will fill it in 5 hours; how long will it take both to fill it?

264. Two pipes together fill a cistern in 6 hours; the first can fill it in 10 hours; how long will it take the second to fill it?

265. Two pipes carry water into a tank, and a third carries water from it. The first pipe will fill it in 2 hours, the second in 3 hours; the third will empty the tank in $1\frac{1}{2}$ hours; if the tank is empty and all 3 pipes are used, in what time will the tank be full?

266. A cistern holding 70 gallons has a pipe by which 15 gallons will run into the cistern in 1 hour, and another that will discharge 10 gallons an hour; when both are running, what part of the cistern will be filled in 3 hours?

267. A can do a piece of work in 3 days; B can do the same work in 4 days; if A earns \$2 a day, what does B earn per day?

268. John is 16 years old, and James is $\frac{3}{4}$ as old; how old is James?

269. In a school there are 27 girls, and $\frac{2}{3}$ as many boys; how many scholars are there in the school?

270. A boy, having 80 marbles, lost $\frac{3}{5}$ and sold $\frac{3}{16}$ of them; how many had he left?

271. A man said that $\frac{3}{4}$ of his money was 4 times his week's wages; he had \$100; what were his week's wages?

272. If a bushel of wheat costs $\$1\frac{2}{3}$, and a bushel of corn $\$\frac{3}{4}$, what is the difference in the cost of 5 bushels of each?

273. John gave away $\frac{2}{3}$ of his marbles, and lost $\frac{3}{4}$ of the remainder; how many had he left if he had 60 at first?

274. In traveling 72 miles a man went $\frac{2}{3}$ of the distance the first day, $\frac{1}{8}$ of the distance the second day, and the remainder the third day; how far did he travel the third day?

275. Of a flock of sheep $\frac{1}{4}$ are in one field, $\frac{1}{6}$ in a second, $\frac{1}{3}$ of the remainder, or 14, in the third; how many sheep are there in the flock?

276. After spending $\frac{1}{5}$ of my money, and losing $\frac{1}{6}$ of the remainder, I have \$30 left; how much had I at first?

277. By selling a watch at a loss of \$36, I lost $\frac{2}{5}$ of its value; what was its value?

278. By selling a watch for \$36, I lost $\frac{2}{5}$ of its value; what was its value?

279. By selling a watch for \$36, I gained $\frac{4}{5}$ of its value; what was its value?

280. A man bought 6 gallons of vinegar at $12\frac{1}{2}$¢ a gallon, and paid for it in oranges at 36¢ a dozen; how many oranges did it take?

281. Walter is $\frac{3}{9}$ as old as his father, and $\frac{3}{8}$ as old as his mother; if he is 18 years old, how old are his father and mother?

282. I bought 100 pounds of sugar at $4\frac{3}{4}¢$ a pound, and paid for it with codfish at $12\frac{1}{2}¢$ a pound; how many pounds of codfish did it take?

283. How many barrels of flour, at $\$6\frac{1}{4}$ a barrel, must be given in exchange for 25 barrels of apples at \$3 a barrel?

284. How many dozen eggs, at $12\frac{1}{2}¢$ a dozen, will pay for 5 pounds of candy at $10¢$ a pound?

285. A person owning $\frac{3}{4}$ of a ship sold $\frac{2}{3}$ of his share for \$5000; what was the value of the ship?

286. If a man can do a piece of work in $12\frac{1}{2}$ days, working 8 hours per day, how many days will it take, working 10 hours a day?

287. If 6 cakes cost $15¢$, what will 7 cakes cost?

288. If $\frac{3}{5}$ of a melon costs $12¢$, how many apples at $2¢$ each will buy the melon?

289. I sold a cow for \$32, which was $\frac{4}{5}$ of her cost; what was the cost?

290. A man sold a cow at a loss of \$16; the loss was $\frac{2}{3}$ of her value; what was her value?

291. If $\frac{5}{6}$ of a sum of money is $\frac{3}{10}$ of the value of a horse, and $\frac{1}{5}$ of the value of the horse is \$20, what is the sum of money?

292. At a selling price of \$18 for sheep, $\frac{1}{4}$ the cost of the sheep was lost; what was the cost?

293. $\frac{2}{3}$ of $\frac{3}{4}$ of 16 is $\frac{5}{6}$ of $\frac{12}{25}$ of how many times 5?

294. A lady bought 10 pounds of raisins at $12\frac{1}{2}¢$ a pound, and paid for them with currants at $5¢$ a pound; how many pounds of currants did it take?

295. If 8 pounds of soda cost $9\frac{3}{4}¢$, what will 5 pounds cost?

296. How many pounds of coffee, at $33\frac{1}{3}¢$ a pound, must be given in exchange for 8 pounds of tea at $66\frac{2}{3}¢$ a pound?

297. A has $\$1\frac{2}{5}$, and B $\$2\frac{1}{5}$; they divide what they both have equally between two persons; how much does each receive?

298. If a river flows $2\frac{5}{8}$ miles in $3\frac{1}{2}$ hours, how far will it flow in 1 hour?

299. $\frac{8}{9}$ is $\frac{4}{5}$ of what number?

300. $\frac{3}{5}$ is $\frac{4}{3}$ of how many times $\frac{9}{10}$?

301. $\frac{3}{8}$ of $3\frac{3}{7}$ is $\frac{9}{10}$ of how many times $\frac{1}{7}$?

302. When cheese is $\frac{9}{50}$ of a dollar a pound, what will $\frac{2}{3}$ of a pound cost?

303. If I buy turkeys at the rate of 5 for $3, and sell at the rate of 8 for $7, how much will I gain on 40 turkeys?

304. How many pigs can I buy for $75, at the rate of 3 for $7, and have $5 left?

305. If 4 men can dig a ditch in 16 days, what part of it will three men dig in 7 days?

306. If a man were twice as old, $\frac{1}{4}$ of his age would be 20 years; how old is he?

307. B gave $\frac{5}{7}$ of all his money for a cow; he paid $12 for hens, which was $\frac{3}{4}$ of all the money he had left; how much had he at first?

308. I bought stock at $800; $\frac{3}{4}$ of this is $\frac{5}{6}$ of $\frac{2}{5}$ of 2 times the present value of the stock; what is its present value?

309. $\frac{4}{7}$ of 85 is $\frac{1}{7}$ of how many times $\frac{2}{3}$ of $\frac{3}{5}$ of 50?

310. $\frac{8}{11}$ is $\frac{2}{3}$ of what number?

311. $\frac{6}{25}$ of $7\frac{6}{17}$ is $\frac{5}{8}$ of what number?

312. Frank is 16 years old; if 4 years were added to his age, he would be $\frac{5}{9}$ as old as his brother; how old is his brother?

313. $\frac{3}{7}$ of $\frac{5}{6}$ of 2×28 is $\frac{5}{9}$ of $\frac{3}{8}$ of what number?

314. $\frac{2}{3}$ the sum of two equal numbers is 20; what are the numbers?

DECIMALS.

The principles of the decimal notation may be extended to certain fractions by placing a period, called a *decimal point*, after units' place, and writing the fraction to the right as lower orders in the decimal scale.

The orders to the right of the decimal point are *tenths*, *hundredths*, *thousandths*, *ten-thousandths*, *hundred-thousandths*, *millionths*, etc., and only those fractions which have these denominators can be written as decimals.

The part to the left of the decimal point is an *integer;* the part to the right, a *decimal fraction.*

The fractional part is not read on the same plan as the integral part.

We read the whole as an integer for the *numerator,* and then declare the denomination of its last digit for the *denominator.*

The first reading represents a fraction plus a fraction; the second, their sum.

ILLUSTRATION.

345.6789

5, *in units' place.*

., *decimal point.*

1 unit = 10 tenths ($\frac{1}{1}\frac{0}{0}$); 1 tenth = 10 hundredths ($\frac{1}{1}\frac{0}{0}\frac{0}{0}$); 1 hundredth = 10 thousandths ($\frac{1}{1}\frac{0}{0}\frac{0}{0}\frac{0}{0}$); etc.

345.6789

345, *integer.*

.6789, *decimal fraction.*

.234567, on the plan of reading integers, would be read 234 *thousandths*, 567 *millionths.*

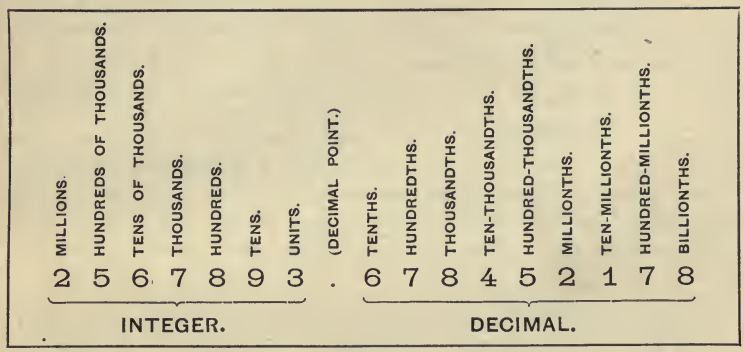
It is actually read, 234 thousand 567 *millionths.*

First. Second.

$\frac{234}{1000} + \frac{567}{1000000} = \frac{234567}{1000000}$.

To write a fraction whose denominator is 10, 100, 1000, etc. (that is, a decimal fraction), we write the numerator in the usual manner, and indicate the denominator by the aid of a decimal point.

$\frac{3}{10} = .3$
$\frac{3}{100} = .03$
$\frac{3}{1000} = .003$



Beginning with the decimal point, numerate:

1. .23456789254.
2. .000240506782.
3. .0000123045123.
4. .365; .4685; .032; .007.
5. .001; .00024; .007058.
6. .20304; .506; .8075.

Read:

7. Ex. 4.
8. Ex. 5.
9. Ex. 6.
10. .0000001.
11. .234$\frac{2}{3}$.
12. 60.005.
13. 800.3065.
14. 40.087.
15. 2683.3.
16. 49675.35.
17. 3.000003.
18. 5.604002.

Ex. **11**. 234$\frac{2}{3}$ thousandths. A common fraction is of the same denomination as the order which it follows.

Ex. **13**. 800 *and* 3 thousand 65 ten-thousandths. In reading mixed numbers, *and* is used only for the decimal point.

§ 25. Reduction — Common Fractions to Decimals.

19. Reduce $\frac{5}{8}$ to a decimal.

$$8)\underline{5.000}$$
$$.625$$

$\frac{5}{8}$ means $5 \div 8$; performing the indicated operation, we obtain .625.

20. Reduce $\frac{2}{7}$ to a decimal.

1st result.

$$7)\underline{2.000}$$
$$.285\tfrac{5}{7}, \text{ or } .285+$$

2d result.

$$7)\underline{2.000000}$$
$$.285714$$

The factors of 10 are 5 and 2; hence, if a fraction has any factor other than 2 and 5 in its denominator, the division will not be exact.

In such cases, the usual plan is to carry out the division a few places and to write the remainder, or to write the sign '+' instead of the remainder.

As a curiosity, the division may be carried out until the quotient begins to repeat, and dots may be placed over the first and last of the figures which repeat, as in 2d result. The part which repeats is then called a *repetend*.

To be memorized:

$\frac{1}{2}=.50$	$\frac{1}{3}=.33\frac{1}{3}$	$\frac{2}{3}=.66\frac{2}{3}$	$\frac{1}{4}=.25$	$\frac{3}{4}=.75$
$\frac{1}{5}=.20$	$\frac{2}{5}=.40$	$\frac{3}{5}=.60$	$\frac{4}{5}=.80$	$\frac{1}{6}=.16\frac{2}{3}$
$\frac{5}{6}=.83\frac{1}{3}$	$\frac{1}{8}=.12\frac{1}{2}$	$\frac{3}{8}=.37\frac{1}{2}$	$\frac{5}{8}=.62\frac{1}{2}$	$\frac{7}{8}=.87\frac{1}{2}$

Give the decimal equivalents rapidly:

21. $\frac{2}{5}, \frac{2}{3}, \frac{1}{8}, \frac{7}{8}, \frac{1}{6}, \frac{3}{4}, \frac{1}{3}, \frac{2}{3}, \frac{5}{8}, \frac{1}{6}$. **23.** $\frac{3}{4}, \frac{1}{2}, \frac{5}{6}, \frac{3}{8}, \frac{1}{6}, \frac{5}{8}, \frac{5}{6}, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}$.

22. $\frac{2}{3}, \frac{7}{8}, \frac{5}{8}, \frac{3}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{8}, \frac{1}{3}, \frac{3}{8}, \frac{4}{5}$. **24.** $\frac{4}{5}, \frac{2}{3}, \frac{1}{4}, \frac{7}{8}, \frac{5}{6}, \frac{3}{8}, \frac{2}{5}, \frac{1}{2}, \frac{1}{5}, \frac{3}{5}$.

Ex. **23.** .75; .50; $.83\frac{1}{3}$; $.37\frac{1}{2}$; etc. Do not look at above table.

Reduce to decimals:

25. $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}$. **27.** $\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{9}{12}, \frac{11}{12}, \frac{1}{13}, \frac{2}{13}$

26. $\frac{5}{9}, \frac{7}{9}, \frac{8}{9}, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}$. **28.** $\frac{4}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{12}{13}, \frac{1}{14}, \frac{3}{14}$.

Ex. **25.** 1st. $.14\frac{2}{7}$; $\frac{1}{7} = 1 \div 7 = .14\frac{2}{7}$.

§ 26. Reduction — Decimals to Common Fractions.

Give the equivalents in common fractions rapidly:

29. .50; .33$\frac{1}{3}$; .87$\frac{1}{2}$.
30. .12$\frac{1}{2}$; .16$\frac{2}{3}$; .33$\frac{1}{3}$.
31. .25; .20; .37$\frac{1}{2}$.
32. .80; .83$\frac{1}{3}$; .62$\frac{1}{2}$.
33. .87$\frac{1}{2}$; .66$\frac{2}{3}$; .40.
34. .75; .20; .60.
35. .83$\frac{1}{3}$; .62$\frac{1}{2}$; .87$\frac{1}{2}$.
36. .75; .33$\frac{1}{3}$; .66$\frac{2}{3}$.

Ex. 29. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$. Having memorized the table in the previous section, the student should read these at a glance.

Reduce to improper fractions:

37. 3.50; 2.75; 3.40.
38. 1.33$\frac{1}{3}$; 3.66$\frac{2}{3}$; 2.12$\frac{1}{2}$.
39. 3.37$\frac{1}{2}$; 2.83$\frac{1}{3}$; 1.16$\frac{2}{3}$.
40. 4.25; 6.87$\frac{1}{2}$; 9.2.
41. 7.50; 9.87$\frac{1}{2}$; 4.62$\frac{1}{2}$.
42. 9.75; 5.83$\frac{1}{3}$; 6.80.
43. 3.16$\frac{2}{3}$; 5.33$\frac{1}{3}$; 8.80.
44. 3.12$\frac{1}{2}$; 1.62$\frac{1}{2}$; 9.30.
45. 2.6; 5.8; 4.66$\frac{2}{3}$.
46. 3.62$\frac{1}{2}$; 5.5; 6.25.
47. 7.2; 8.75; 9.6.
48. 5.66$\frac{2}{3}$; 7.83$\frac{1}{3}$; 6.87$\frac{1}{2}$.
49. 8.50; 5.62$\frac{1}{2}$; 6.83$\frac{1}{3}$.
50. 9.7; 8.62$\frac{1}{2}$; 7.33$\frac{1}{3}$.
51. 6.66$\frac{2}{3}$; 7.90; 5.16$\frac{2}{3}$.
52. 8.80; 7.33$\frac{1}{3}$; 4.83$\frac{1}{3}$.

Ex. 39. $\frac{27}{8}$. $3.37\frac{1}{2} = 3\frac{3}{8} = \frac{27}{8}$.

Reduce to common fractions:

53. .48; .25; .375.
54. .960; .225; .144.
55. .625; .200; .16.
56. .516; .750; .90.
57. .58; .265; .735.
58. .875; .3125; .4375.
59. .82$\frac{1}{2}$; .72$\frac{1}{2}$; .6$\frac{2}{3}$.
60. .7$\frac{1}{4}$; .20$\frac{1}{5}$; .18$\frac{1}{3}$.

Ex. 53. 1st. $\frac{12}{25}$. $.48 = \frac{48}{100} = \frac{12}{25}$.

Ex. 59. 1st. $\frac{33}{40}$. $.82\frac{1}{2} = \frac{165}{200} = \frac{33}{40}$.

§ 27. Per Cent.

When the denominator is 100, there are three ways of expressing the fraction.

1st. As a *common fraction.*

2d. As a *decimal fraction.*

3d. By the *per cent* symbol.

Per cent and *hundredths* are interchangeable.

Illustration.

Six hundredths may be written,

$\frac{6}{100}$, *a common fraction,*

.06, *a decimal fraction,*

6%, read 6 *per cent.*

8% = .08.

Change to %:

61. .06; .08; .09.

62. 500 hundredths.

63. $\frac{14}{100}$; $.00\frac{1}{2}$; $\frac{\frac{4}{5}}{100}$.

64. .16; .85; .96.

65. 4000 hundredths.

66. 8365 hundredths.

Ex. **62.** 500%. Ex. **66.** 8365%.

Change to decimals:

67. 5%; 8%; 23%.

68. 233%; 4025%.

69. 86%; 75%.

70. $\frac{1}{2}$%; $\frac{2}{3}$%.

71. $.000\frac{3}{4}$%; 2%.

72. 200%; 65%.

Ex. **68.** 2.33; 40.25.

Give the equivalents as %:

73. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

74. $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{3}{16}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{3}{6}$.

75. $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$, $5\frac{5}{6}$, $6\frac{7}{8}$, $8\frac{1}{4}$, $9\frac{3}{8}$.

Ex. **73.** 50%, $33\frac{1}{3}$%, etc. See p. 74.

Give the equivalents as fractions:

76. $87\frac{1}{2}$%, $66\frac{2}{3}$%, 80%, 50%, $62\frac{1}{2}$%, $16\frac{2}{3}$%, 20%, 60%.

77. $16\frac{2}{3}$%, $37\frac{1}{2}$%, $87\frac{1}{2}$%, $83\frac{1}{3}$%, 40%, 75%, 25%, $33\frac{1}{3}$%.

78. $183\frac{1}{3}$%, 280%, $137\frac{1}{2}$%, 125%, $233\frac{1}{3}$%, 175%, 225%.

79. $116\frac{1}{2}$%, 220%, 140%, 375%, $433\frac{1}{3}$%, $212\frac{1}{2}$%, $416\frac{2}{3}$%.

Ex. **76.** $\frac{1}{8}$, $\frac{2}{3}$, $\frac{4}{5}$, etc. See p. 74.

§ 28. Short Methods.

To multiply when the sum of the fractional parts is *one*, and the integers are the same.

To the product of the integer and the integer increased by one, annex the product of the fractions.

79. What is $6\frac{3}{4} \times 6\frac{1}{4}$?

Illustration.

$6\frac{3}{4}$
$6\frac{1}{4}$

$\frac{1}{4} \times \frac{3}{4}$
$6 \times \frac{1}{4}$
$6 \times \frac{3}{4}$
6×6

$6 \times 7 + \frac{1}{4} \times \frac{3}{4} = 42\frac{3}{16}$.

Find the value of:

80.	$6\frac{1}{2} \times 6\frac{1}{2}$.	**96.**	$1\frac{1}{4} \times 1\frac{3}{4}$.	**112.**	8.3×8.7.
81.	$7\frac{1}{2} \times 7\frac{1}{2}$.	**97.**	$9\frac{1}{8} \times 9\frac{7}{8}$.	**113.**	9.2×9.8.
82.	$8\frac{1}{2} \times 8\frac{1}{2}$.	**98.**	$6\frac{5}{8} \times 6\frac{3}{8}$.	**114.**	7.2×7.8.
83.	$4\frac{3}{4} \times 4\frac{1}{4}$.	**99.**	$9\frac{1}{6} \times 9\frac{5}{6}$.	**115.**	2.2×2.8.
84.	$6\frac{1}{7} \times 6\frac{6}{7}$.	**100.**	$8\frac{1}{3} \times 8\frac{2}{3}$.	**116.**	2.3×2.7.
85.	$5\frac{1}{5} \times 5\frac{4}{5}$.	**101.**	$5\frac{2}{5} \times 5\frac{3}{5}$.	**117.**	85×85.
86.	$9\frac{3}{7} \times 9\frac{4}{7}$.	**102.**	$8\frac{5}{9} \times 8\frac{4}{9}$.	**118.**	74×76.
87.	$3\frac{2}{3} \times 3\frac{1}{3}$.	**103.**	$7\frac{3}{11} \times 7\frac{8}{11}$.	**119.**	83×87.
88.	$8\frac{4}{5} \times 8\frac{1}{5}$.	**104.**	8.5×8.5.	**120.**	92×98.
89.	$3\frac{1}{8} \times 3\frac{7}{8}$.	**105.**	7.4×7.6.	**121.**	27×23.
90.	$5\frac{1}{2} \times 5\frac{1}{2}$.	**106.**	9.3×9.7.	**122.**	38×32.
91.	$10\frac{1}{2} \times 10\frac{1}{2}$.	**107.**	4.8×4.2.	**123.**	54×56.
92.	$11\frac{1}{2} \times 11\frac{1}{2}$.	**108.**	11.9×11.1.	**124.**	31×39.
93.	$99\frac{1}{2} \times 99\frac{1}{2}$.	**109.**	5.7×5.3.	**125.**	63×67.
94.	$10\frac{1}{12} \times 10\frac{11}{12}$.	**110.**	4.6×4.4.	**126.**	58×52.
95.	$12\frac{1}{13} \times 12\frac{12}{13}$.	**111.**	10.9×10.1.	**127.**	4.75×4.25.

Ex. **80.** 42.25. $6 \times (6 + 1) = 42$; $.5 \times .5 = .25$.
Ex. **117.** 7225. $85 \times 85 = 8.5 \times 8.5 \times 100 = 7225$.

To multiply or divide when one number can be readily reduced to a simple fraction.

Reduce to the fraction and multiply.

ILLUSTRATION.

$.66\frac{2}{3}$ of $24 = \frac{2}{3}$ of $24 = 16$.

Multiply:

128. 24 by .50.
129. 36 by .75.
130. 84 by $.83\frac{1}{3}$.
131. 72 by $.87\frac{1}{2}$.
132. 33 by $.33\frac{1}{3}$.
133. 24 by $.66\frac{2}{3}$.
134. 56 by $.87\frac{1}{2}$.
135. 36 by $.83\frac{1}{3}$.
136. 25 by .60.
137. 75 by $.33\frac{1}{3}$.
138. 21 by $.66\frac{2}{3}$.
139. 48 by $.37\frac{1}{2}$.
140. 23 by $.33\frac{1}{3}$.
141. 26 by $.66\frac{2}{3}$.
142. 27 by .25.
143. 29 by .50.
144. 30 by $.12\frac{1}{2}$.
145. 31 by $.37\frac{1}{2}$.

Ex. **129.** 27. $.75 = \frac{3}{4}$; $36 \times \frac{3}{4} = 27$.

What is:

146. .50 of 16? $.87\frac{1}{2}$ of 40?
147. $.33\frac{1}{3}$ of 60? $.62\frac{1}{2}$ of 64?
148. $.66\frac{2}{3}$ of 36? $.37\frac{1}{2}$ of 56?
149. .25 of 52? $.12\frac{1}{2}$ of 72?
150. .40 of 32? .60 of 13?
151. $.16\frac{2}{3}$ of 25? $.33\frac{1}{3}$ of 19?
152. .20 of 53? .25 of 35?
153. $.37\frac{1}{2}$ of 22? $1.12\frac{1}{2}$ of 16?
154. 20% of 50? 75% of 16?
155. $66\frac{2}{3}$% of 72? 25% of 28?
156. $16\frac{2}{3}$% of 42? $33\frac{1}{3}$% of 18?
157. 40% of 60? $37\frac{1}{2}$% of 16?
158. 60% of 20? $62\frac{1}{2}$% of 24?
159. $183\frac{1}{3}$% of 30? $116\frac{2}{3}$% of 36?
160. $212\frac{1}{2}$% of 40? 325% of 16?
161. $16\frac{2}{3} \times 18$? 25×120?
162. 75×96? $37\frac{1}{2} \times 64$?
163. $66\frac{2}{3} \times 48$? $83\frac{1}{3} \times 28$?
164. 75×40? 25×80?

Ex. **159.** 55. $183\frac{1}{3}\% = \frac{11}{6}$; $\frac{11}{6}$ of $30 = 55$.
Ex. **161.** 300. $16\frac{2}{3} \times 18 = .16\frac{2}{3} \times 18 \times 100 = 300$.

What is the cost of:

165. 120 yards of cloth at 50¢ a yard? $33\frac{1}{3}$¢? 25¢?
166. 12 yards of cloth at $\$1.16\frac{2}{3}$ a yard? $2.75? $3.25?
167. 25 yards of cloth at 25¢ a yard? $12\frac{1}{2}$¢? $87\frac{1}{2}$¢?
168. 72 dolls at 25¢ each? $33\frac{1}{3}$¢? $37\frac{1}{2}$¢? 50¢? $62\frac{1}{2}$¢?
169. 72 chairs at $83\frac{1}{3}$¢ each? $87\frac{1}{2}$¢? 20¢? 40¢? 60¢?
170. 24 lamps at $\$1.37\frac{1}{2}$ a dozen? $2.75 a dozen?
171. 60 chimneys at 40¢ a dozen? $37\frac{1}{2}$¢ a dozen?
172. 9 vests at $8.75 a dozen? $3.25 a dozen?
173. 24 spools of thread at $2.75 per 100? $4.50 a gross?
174. 16 pounds of ham at $8\frac{1}{3}$¢ per pound? $12\frac{1}{2}$¢? $16\frac{2}{3}$¢?
175. 3 dozen lemons at 6 for $12\frac{1}{2}$¢? 8 for a quarter?
176. 48 lamp chimneys at $33\frac{1}{3}$¢ per dozen?
177. 40 tables at $1.25 each? $1.75? $\$1.66\frac{2}{3}$? $\$1.87\frac{1}{2}$?
178. 18 table-cloths at $9 a dozen? $12.50 a dozen?
179. 64 hammocks at $1.25 each? $1.50? $\$1.83\frac{1}{3}$? $1.75?
180. 28 oranges at 25¢ per dozen? 50¢ per dozen?
181. 45 bananas at 75¢ per dozen? $33\frac{1}{3}$¢ per dozen?
182. 15 plums at 20¢ per dozen? $12\frac{1}{2}$¢ per dozen?
183. 30 knives at 60¢ each? 75¢ each? 80¢ each?
184. 20 bottles of ink at 60¢ per dozen? 30¢ per dozen?
185. 12 dozen pens at $16\frac{2}{3}$¢ per dozen? 25¢ per dozen?
186. 100 cloaks at $\$12\frac{1}{2}$ each? $50? $75? $20?
187. 21 pair of shoes at $2.75 a pair? $3.25? $5? $2.50?
188. 4 dozen plates at $1.25 per dozen? at $12\frac{1}{2}$¢ each?
189. 8 dozen cups at $\$4.37\frac{1}{2}$ per dozen? $3.40 per dozen?
190. 60 marbles at 10¢ per dozen? 25¢ per dozen?
191. 80 chains at 75¢ each? 50¢? $2.25? $\$3.66\frac{2}{3}$? $4.40?
192. 50 glasses at 25¢ each? $33\frac{1}{3}$¢? $87\frac{1}{2}$¢? $37\frac{1}{2}$¢?
193. 75 baskets at $12\frac{1}{2}$¢ each? $16\frac{2}{3}$¢? 20¢? 25¢? 75¢?

Ex. **165.** $60; $40; etc.

Find the value of:

194. $24 \div .66\frac{2}{3}$.
195. $24 \div 66\frac{2}{3}\%$.
196. $24 \div 66\frac{2}{3}$.
197. $210 \div .50$.
198. $210 \div .33\frac{1}{3}$.
199. $210 \div 66\frac{2}{3}\%$.
200. $210 \div 75$.
201. $210 \div 87\frac{1}{2}$.
202. $210 \div .25$.
203. $210 \div 25\%$.
204. $210 \div 25$.
205. $210 \div 83\frac{1}{3}$.

Ex. **194.** 36. Ex. **195.** 36.
Ex. **196.** $\frac{9}{25}$. $24 \div 66\frac{2}{3} = (24 \div \frac{2}{3}) \div 100 = \frac{36}{100} = \frac{9}{25}$.

What is:

206. $21 \div 60\%$? By $37\frac{1}{2}\%$?
207. $21 \div 50\%$? By 75%?
208. $21 \div 87\frac{1}{2}\%$? By $16\frac{2}{3}\%$?
209. $20 \div 66\frac{2}{3}\%$? By $33\frac{1}{3}\%$?
210. $20 \div 40\%$? By $62\frac{1}{2}\%$?
211. $20 \div 83\frac{1}{3}\%$? By 50%?
212. $25 \div 16\frac{2}{3}\%$? By $33\frac{1}{3}\%$?
213. $25 \div 83\frac{1}{3}\%$? By $12\frac{1}{2}\%$?
214. $33 \div 150\%$? By $137\frac{1}{2}\%$?
215. $42 \div 140\%$? By 175%?
216. $66 \div 183\frac{1}{3}\%$? By 75%?
217. $1225 \div 25\%$? By $33\frac{1}{3}$?
218. $213 \div 25\%$? By 50%?
219. $472 \div 25\%$? By $66\frac{2}{3}\%$?
220. $230 \div 25\%$? By $16\frac{2}{3}\%$?
221. $450 \div 25\%$? By $12\frac{1}{2}\%$?

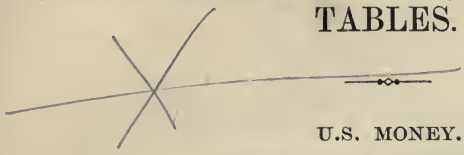
Ex. **217.** 4900. $1225 \div .25 = 1225 \times 4 = 4900$.

How many yards can be bought:

222. For \$12 at 25¢? 75¢? $16\frac{2}{3}$¢? $66\frac{2}{3}$¢? $33\frac{1}{3}$¢? $37\frac{1}{2}$¢?
223. For \$12 at 60¢? $87\frac{1}{2}$¢? $83\frac{1}{3}$¢? $12\frac{1}{2}$¢? 50¢? $62\frac{1}{2}$¢?
224. For \$120 at \$1.25? \$1.33$\frac{1}{3}$? \$1.66$\frac{2}{3}$? \$1.50?
225. For \$120 at \$1.87$\frac{1}{2}$? \$3.75? \$2.50? \$1.60?
226. For \$150 at \$1.20? \$1.25? \$3.33$\frac{1}{3}$? \$.75?
227. For \$150 at \$2.50? \$1.87$\frac{1}{2}$? \$1.66$\frac{2}{3}$? \$1.33$\frac{1}{3}$?
228. For \$150 at \$.12$\frac{1}{2}$? \$3.16$\frac{2}{3}$? \$.33$\frac{1}{3}$? \$1.25?
229. For \$140 at \$1.60? \$1.20? \$12$\frac{1}{2}$? \$16$\frac{2}{3}$?
230. For \$140 at \$25? \$37$\frac{1}{2}$? \$1.87$\frac{1}{2}$? \$1.75?

Ex. **224.** 96 yards. $120 \div 1.25 = 120 \div \frac{5}{4} = 96$.

DENOMINATE NUMBERS — ENGLISH TABLES.



U.S. MONEY.

10 mills (m.)	= 1 cent (¢)
10 ¢	= 1 dime (d.)
10 d.	= 1 dollar ($)
$10	= 1 eagle (E.)

ENGLISH MONEY.

4 farthings (far. or qr.)	= 1 penny (*d.*)
12 *d.*	= 1 shilling (*s.*)
20 *s.*	= 1 pound (£)
21 *s.*	= 1 guinea (G.)

The table of U.S. money, like the tables of the Metric system, has the submultiples: Latin *mille*, 1000, *mill;* Latin *centum*, 100, *cent;* Latin *decem*, 10, *dime*. *Ten* of any denomination always make *one* of the next higher.

A sovereign = £1.

In English money, the symbols £, *s.*, *d.*, qr., are the initials of the Latin equivalents, *libra*, *solidus*, *denarius*, and *quadrans*.

TROY WEIGHT.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pwt.	= 1 ounce (oz.)
12 oz.	= 1 pound (lb.)

AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	= 1 pound (lb.)
100 lb.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton (T.)

APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple (℈)
3 ℈	= 1 dram (ʒ)
8 ʒ	= 1 ounce (℥)
12 ℥	= 1 pound (℔)

APOTHECARIES' FLUID MEASURE.

60 minims (♏)	= 1 fluid drachm (f ʒ)
8 f ʒ	= 1 fluid ounce (f ℥)
16 f ℥	= 1 pint (O.)
8 O.	= 1 gallon (Cong.)

Troy weight is used in weighing gold, silver, and precious stones; avoirdupois weight, in weighing nearly everything else; apothecaries' weight, in retailing dry medicines; apothecaries' fluid measure, in retailing liquid medicines.

1 long ton = 2240 lb.; 1 carat = 4 gr. *Carat* is also used to denote the number of parts in 24 that are pure gold. *E.g.* '12 carats fine' means $\frac{12}{24}$ gold.

Grain is from grain of wheat; 7000 gr. = 1 lb. avoirdupois weight, 5760 gr. = 1 lb. troy or apothecaries' weight. *Pennyweight* was the weight of the English penny in silver; *ounce*, Latin *uncia*, *twelfth*; *pound*, Latin *pendo*, weigh; *scruple*, Latin *scrupulus*, a little stone; *dram*, Greek, *drachma*. *Cong.* is from the Latin *congius*, a gallon; O is the abbreviation for Latin *octavus*, one eighth, the pint being one eighth of a gallon. The origin of symbols of apothecaries' weight is unknown.

LONG MEASURE.

12 inches (in.)	= 1 foot (ft.)
3 ft.	= 1 yard (yd.)
$5\frac{1}{2}$ yd. or $16\frac{1}{2}$ ft.	= 1 rod (rd.)
320 rd.	= 1 mile (mi.)

SURVEYORS' LONG MEASURE.

7.92 inches (in.)	= 1 link (li.)
100 li.	= 1 chain (ch.)
66 ft.	= 1 chain.
80 ch.	= 1 mile (mi.)

Long measure is used in measuring lengths and distances. For short lengths, the foot rule or yard stick is commonly used; for long distances, a chain 4 rods long. The *surveyor's* chain was made 4 rods in order that 10 square chains might equal 1 acre. It is divided for convenience into 100 links; 4 rd. = 66 ft. = 792 in.; hence, 7.92 in. = 1 li. The *engineer's* chain is 100 feet long.

Inch is from Latin *uncia*, twelfth; *foot*, human foot; *yard*, a twig; *mile*, Latin *mille passuum*, 1000 paces.

A line = $\frac{1}{12}$ in.; a furlong = 40 rd.; a fathom = 6 ft.

SQUARE MEASURE.

144 square inches (sq. in.) = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
$30\frac{1}{4}$ sq. yd. or $272\frac{1}{4}$ sq. ft. = 1 sq. rd.
160 sq. rd. = 1 acre (A.)

SURVEYORS' SQUARE MEASURE.

10000 sq. li. = 1 sq. ch.
10 sq. ch. = 1 A.
640 A. = 1 sq. mi.
36 sq. mi. = 1 township (Tp.)

Square measure is used in stating *areas* and *surfaces;* it is formed by squaring long measure; 12 in. = 1 ft.; squaring, 144 sq. in. = 1 sq. ft.; 3 ft. = 1 yd., squaring, 9 sq. ft. = 1 sq. yd.; etc.

A perch = 1 sq. rd.; a rood = 40 sq. rd.

CUBIC MEASURE.

1728 cubic inches (cu. in.) = 1 cu. ft.
27 cu. ft. = 1 cu. yd.

WOOD MEASURE.

16 cu. ft. = 1 cord foot (cd. ft.)
8 cd. ft. = 1 cord (cd.)

Cubic measure is used in stating *volumes* or *contents;* it is formed by cubing long measure; 12 in. = 1 ft., cubing, 1728 cu. in. = 1 cu. ft.; 3 ft. = 1 yd., cubing, 27 cu. ft. = 1 cu. yd.

CAPACITY.

LIQUID MEASURE.	DRY MEASURE.
4 gills (gi.) = 1 pint (pt.)	2 pt. = 1 qt.
2 pt. = 1 quart (qt.)	8 qt. = 1 peck (pk.)
4 qt. = 1 gallon (gal.)	4 pk. = 1 bushel (bu.)

Liquid measure is used for measuring liquids; dry measure, for measuring dry substances.

Gill is from Latin *gilla*, a drinking-glass; *pint*, Spanish *pinta*, a mark; *quart*, Latin *quartus*, a fourth; *gallon*, derivation unknown.

For convenience, the number of bushels of grain and seed is determined by weighing instead of by using a bushel measure. Many of the states have fixed by statute the number of pounds to be reckoned a bushel. See p. 86.

The barrel does not contain any uniform number of gallons; the number of gallons is usually written on one of its heads. 31½ gallons is the standard.

TIME.

60 seconds (sec.)	= 1 minute (min.)
60 min.	= 1 hour (h.)
24 h.	= 1 day (da.)
7 da.	= 1 week (wk.)
4 wk.	= 1 month (mo.)
12 mo.	= 1 year (yr.)
365 da.	= 1 common yr.
366 da.	= 1 leap yr.
100 yr.	= 1 century (cen.)

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine."

Every year exactly divisible by 4, centennial years excepted, is a leap year. — 1892, a leap year. 1893, a common year.

Every centennial year exactly divisible by 400 is a leap year; the others are common years. — 2000, a leap year. 1900, a common year.

How many days are there in Mar.? Nov.? June? Aug.? Sept.? Feb.? Apr.? Jan.? May? July? Oct.? Dec.?

What month numerically is Aug.? June? Nov.? Mar.? Dec.? Oct.? July? May? Jan.? Apr.? Feb.? Sept.?

A *day* is the time of the revolution of the earth upon its axis; a *month* is the time of the revolution of the moon around the earth. The months are: January (31 days), February (28 or 29), March (31), April (30), May (31), June (30), July (31), August (31), September (30), October (31), November (30), December (31). The months are also designated by the ordinal numerals, January being the 1st month.

In business, 30 days are counted as a month.

A *year* is the time required for the earth to revolve around the sun. It is 365 days 5 h. 48 min. 49.7 sec., or very nearly $365\frac{1}{4}$ days. Instead of reckoning this part of a day each year, it is disregarded, and an addition is made when it amounts to *one day*, which occurs about every fourth year. This addition of one day is made to the month of February. Since the part of a day that is disregarded when 365 days are considered as a year, is a little less than one quarter of a day, the addition of one day every fourth year is a little too much, and, to correct this excess, addition is made to only every fourth centennial year. This is why every year divisible by 4 *except centennial years* is a leap year.

Formerly time was reckoned very inaccurately. In 46 B.C. Julius Cæsar reformed the calendar and made the year consist of $365\frac{1}{4}$ days, but even this was not absolutely accurate, and in 1582 the error in the calendar established by him (called the Julian calendar) had increased to 10 days; that is, too much time had been reckoned as a year, so that the civil year was 10 days behind the solar year. To correct this error, Pope Gregory XIII. ordered 10 days to be stricken from the calendar. The day after the 3d of October, 1582, was called the 14th, and thereafter only those centennial years which were divisible by 400 were considered as leap years.

CIRCULAR MEASURE.

60 seconds (″) = 1 minute (′)
60′ = 1 degree (°)
30° = 1 sign (S.)
90° = 1 right angle.
12 S., or 360° = 1 circumference (C.)

Circular measure is used in measuring *angles*. The division of a circumference into 360 parts may have been suggested by the days in a year.

COUNTING.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
480 sheets = 1 ream.

EQUIVALENTS.

231 cu. in. = 1 gal.
2150.4 cu. in. = 1 bu.
4 bu. = 5 cu. ft. (nearly).
4 heaped bu. = 5 struck bu.
$7\frac{1}{2}$ gal. = 1 cu. ft. (nearly).
$62\frac{1}{2}$ lb. = wt. 1 cu. ft. water.
$(7 \text{ ft.})^3$ to $(8 \text{ ft.})^3$ = 1 ton hay.
7 cu. ft. corn in ear = 3 bu. shelled corn.
24 h. = 360°.
60 ♏ = 1 teaspoonful.
5760 gr. = 1 lb. troy.
5760 gr. = 1 lb. apothecaries'.
7000 gr. = 1 lb. avoirdupois.

In most states:

48 lb. = 1 bu. barley.
56 lb. = 1 bu. corn.
32 lb. = 1 bu. oats.
60 lb. = 1 bu. potatoes.
56 lb. = 1 bu. rye.
60 lb. = 1 bu. wheat.

§ 29. Exercises in English Tables.

U.S. MONEY.

How many:

1. m. make $1?
2. d. make $1?
3. ¢ make $1?
4. ¢ make 1 E.?
5. d. make 1 E.?

How many:

6. m. make 1 d.?
7. m. make 1 E.?
8. m. make 1 ¢?
9. ¢ make 1 d.?
10. $ make 1 E.?

ENGLISH MONEY.

How many:

11. far. make £1?
12. *s.* make £1?
13. *s.* make 1 G.?
14. far. make 1 *s.*?
15. far. make 1 G.?

How many:

16. *d.* make 1 *s.*?
17. *d.* make 1 G.?
18. *d.* make £1?
19. far. make 1 *d.*?
20. £ make 1 G.?

TROY AND AVOIRDUPOIS WEIGHTS.

How many:

21. gr. make 1 lb. (troy)?
22. gr. make 1 oz. (troy)?
23. oz. make 1 lb. (troy)?
24. pwt. make 1 lb.?
25. pwt. make 1 oz.?

How many:

26. gr. make 1 lb. (apoth.)?
27. oz. make 1 lb. (apoth.)?
28. cwt. make 1 T.?
29. lb. make 1 T.?
30. gr. make 1 T.?

APOTHECARIES' WEIGHT AND APOTHECARIES' FLUID MEASURE.

How many:

31. ℥ make 1 ƒ℥?
32. ƒ℥ make 1 ƒ℥?
33. ƒ℥ make 1 O.?
34. ƒ℥ make 1 Cong.?
35. ƒ℥ make 1 Cong.?

How many:

36. gr. make 1 ℔?
37. gr. make 1 ℥?
38. ℈ make 1 ℔?
39. ℥ make 1 ℔?
40. ℥ make 1 ℥?

LONG AND SURVEYORS' LONG MEASURE.

How many:

41. in. make 1 yd.?
42. in. make 1 rd.?
43. ft. make 1 rd.?
44. ft. make 1 mi.?
45. rd. make 1 mi.?

How many:

46. li. make 1 ch.?
47. li. make 1 rd.?
48. ch. make 1 rd.?
49. ch. make 1 mi.?
50. in. make 1 li.?

SQUARE AND SURVEYORS' SQUARE MEASURE.

How many:

51. sq. in. make 1 sq. ft.?
52. sq. in. make 1 sq. yd.?
53. sq. ft. make 1 sq. rd.?
54. sq. rd. make 1 A.?
55. sq. ft. make 1 sq. yd.?

How many:

56. sq. ch. make 1 A.?
57. sq. li. make 1 sq. ch.?
58. sq. li. make 1 sq. rd.?
59. A. make 1 sq. mi.?
60. sq. mi. make 1 Tp.?

CUBIC AND ~~WOOD~~ MEASURES.

How many:

61. cu. in. make 1 cu. ft.?
62. cu. in. make 1 cu. yd.?
63. cu. ft. make 1 cu. yd.?
64. cu. in. make 1 cd. ft.?
65. cu. in. make 3 cu. ft.?

How many:

66. cu. ft. make 1 cd.?
67. cu. ft. make 1 cd. ft.?
68. cu. ft. make 3 cu. yd.?
69. cu. ft. make 2 cu. yd.?
70. cu. ft. make 3 cd.?

LIQUID AND DRY MEASURES.

How many:

71. pt. make 1 qt. (dry)?
72. qt. make 1 pk.?
73. pt. make 1 bu.?
74. pt. make 1 pk.?
75. pk. make 1 bu.?

How many:

76. pt. make 1 qt. (liquid)?
77. qt. make 1 gal.?
78. qt. make 1 bbl. of 31½ gal.
79. pt. make 1 bbl. of 45 gal.?
80. qt. make 1 bbl. of 40 gal.?

MISCELLANEOUS.

How many:

81. da. make 1 leap year?
82. ~~degrees make 1 C.~~?
83. cu. in. make 1 gal.? p86
84. ~~cu. in. make 1 bu.~~?
85. ~~degrees make 1 S.?~~

How many:

86. units make 1 dozen?
87. units make 1 score?
88. sheets make 1 quire? p86
89. sheets make 1 ream?
90. units make 1 gross?

§ 30. Reduction — In the Same Table.

Reduce:

91. £3 to *d.*

92. 14 *s.* to far.

93. 5 G. to *s.*

94. £2 3 *s.* 4 *d.* to far.

95. 3 G. 5 *s.* 6 *d.* to *d.*

96. 48 *d.* to *s.*

97. 300 far. to *s.*

98. 3000 far. to £.

99. $\frac{2}{5}$ *s.* to *d.*

100. £$\frac{2}{3}$ to *d.*

Ex. **95.** 822 *d.* 3 G. = 63 *s.*; 68 *s.* = 816 *d.*; 816 *d.* + 6 *d.* = 822 *d.*

Reduce:

101. 2 lb. 3 oz. 2 pwt. (troy) to pwt.

102. 960 gr. (troy) to lb.

103. $\frac{2}{5}$ ℔. (apoth.) to lower integers.

104. $\frac{5}{6}$ lb. (avoir.) to lower integers.

105. $\frac{2}{5}$ T. (avoir.) to oz.

106. ~~8 Cong. to O.~~

107. ~~2 Cong. 3 O. 2 ƒ℥ to ƒ℥.~~

108. 1440 ℈ to ℔.

109. ~~1 Cong. to ♏.~~

110. ~~3600 ♏ to ƒ℥.~~

Ex. **103.** 4 ℥ 6 ℨ 1 ℈ 4 gr.; $\frac{2}{5}$ ℔ = 4$\frac{1}{5}$ ℥; $\frac{4}{5}$ ℥ = 6$\frac{2}{5}$ ℨ; $\frac{2}{5}$ ℨ = 1$\frac{1}{5}$ ℈; $\frac{1}{5}$ ℈ = 4 gr.

Reduce:

111. 3 yd. 2 ft. 3 in. to in.

112. 3 yd. 2 ft. 3 in. to ft.

113. 3 yd. 2 ft. 3 in. to yd.

114. ~~8 ch. 16 li. to li.~~

115. 160 cu. ft. to cd.

116. 1728 in. to rd.

117. $\frac{2}{9}$ rd. to lower integers.

118. 5 sq. rd. to sq. ft.

119. 1 A. to sq. yd.

120. 3500 cu. in. to higher integers.

121. 5184 cu. in. to cu. ft.

Ex. **120.** 2 cu. ft. 44 cu. in.

Reduce:

122. .6 bu. to lower integers.

123. 96 qt. to bu.

124. 2 bu. 2 pk. 1 qt. to lower integers.

125. 252 gal. to pt.

126. $1\frac{1}{2}$ gal. to qt.

127. 2000 pt. to higher integers.

128. $\frac{5}{7}$ gal. to lower integers.

129. 1728 pt. to higher integers.

130. 1728 pt. to gal.

Ex. **122.** 2 pk. 3 qt. .4 pt.; .6 bu. = 2.4 pk.; .4 pk. = 3.2 qt.; .2 qt. = .4 pt.

Reduce:

131. 2 mo. 16 da. to da.

132. 1 leap yr. 2 da. to wk.

133. 2500 min. to higher integers.

134. 2 h. 2 min. 2 sec. to sec.

135. $\frac{5}{7}$ h. to lower integers.

136. 5 score to units.

137. 6 score to dozen.

138. 5 gross to dozen.

139. 36 gross to great gross.

140. 960 sheets to reams.

141. 1200 sheets to reams.

142. 1200 quires to reams.

143. 3° 4′ 5″ to ′.

Ex. **143.** $184\frac{1}{12}'$; $3° = 180'$; $5'' = \frac{1}{12}'$; $180' + 4' + \frac{1}{12}' = 184\frac{1}{12}'$.

Reduce:

144. £1 1*s.* 1*d.* 1 far. to far.

145. $\frac{3}{5}$℔ to lower integers.

146. 7 lb. (troy) to lower integers.

147. 1925 oz. (avoir.) to higher integers.

148. 480 ℥ to ʒ.

149. 1 bu. 1 pk. 1 qt. 1 pt. to pk.

150. 63 gal. to bbl.

151. 33 ft. to a fraction of a mi.

152. 1 A. to sq. yd.

153. 1 cu. yd. to a fraction of a cd.

Ex. **151.** $\frac{1}{160}$ mi.; 33 ft. = 2 rd. = $\frac{2}{320}$ mi. = $\frac{1}{160}$ mi.

§ 31. Reduction — Table to Table.

For most computations, we may consider 4 bu. equal to 5 cu. ft.

4 bu. $= 2150.4 \times 4 = 8601.6$ cu. in.
5 cu. ft. $= 1728 \times 5 = 8640$ cu. in.

For most computations, we may consider $7\frac{1}{2}$ gal. equal to 1 cu. ft.

$7\frac{1}{2}$ gal. $= 231 \times 7\frac{1}{2} = 1732\frac{1}{2}$ cu. in.
1 cu. ft. $= 1728$ cu. in.

Reduce:

154. 20 cu. ft. to bu.
155. 20 bu. to cu. ft.
156. 400 bu. to cu. ft.
157. 400 cu. ft. to bu.
158. 32 bu. to cu. ft.
159. 100 cu. ft. to heaped bu.
160. 80 heaped bu. to cu. ft.
161. 64 heaped bu. to cu. ft.
162. 75 cu. ft. to heaped bu.
163. 8 ft. $\times$ 8 ft. $\times$ 3 ft. to bu.
164. 7 ft. $\times$ 10 ft. $\times$ 2 ft. to bu.
165. 4300.8 cu. in. to bu.
166. 6451.2 cu. in. to bu.
167. 4 bu. to cu. ft.
168. 1 qt. (dry) to cu. in.
169. 1 pt. (dry) to cu. in.
170. 60 cu. ft. to gal.
171. 60 gal. to cu. ft.
172. 15 cu. ft. to gal.
173. 15 gal. to cu. ft.
174. 105 cu. ft. to gal.
175. 105 gal. to cu. ft.
176. 90 cu. ft. to gal.
177. 90 gal. to cu. ft.
178. 25 cu. ft. to gal.
179. 3 ft. $\times$ 5 ft. $\times$ 6 ft. to gal.
180. 6 ft. $\times$ 10 ft. $\times$ 4 ft. to gal.
181. 462 cu. in. to gal.
182. 1155 cu. in. to gal.
183. 4 gal. to cu. in.
184. 1 qt. (liquid) to cu. in.
185. 1 pt. (liquid) to cu. in.

Ex. **154.** 16 bu.; 1 cu. ft. $= \frac{4}{5}$ bu.; 20 cu. ft. $= 20 \times \frac{4}{5}$ bu. $= 16$ bu.

Ex. **155.** 25 cu. ft.; 1 bu. $= \frac{5}{4}$ cu. ft.; 20 bu. $= 20 \times \frac{5}{4}$ cu. ft. $= 25$ cu. ft.

Ex. **160.** 125 cu. ft.; 1 h. bu. $= \frac{5}{4}$ bu.; 1 h. bu. $= \frac{25}{16}$ cu. ft.; 80 h. bu. $= 80 \times \frac{25}{16}$ cu. ft. $= 125$ cu. ft.

Reduce:

186. 5000 gal. water to cu. ft.
187. 10 cu. ft. water to lb.
188. 125 lb. water to cu. ft.
189. 6 cu. ft. water to lb.
190. 720 lb. wheat to bu.
191. 144 lb. barley to bu.
192. 504 lb. rye to bu.
193. 336 lb. corn to bu.
194. 480 lb. potatoes to bu.
195. 640 lb. oats to bu.

Reduce:

196. 686 cu. ft. hay to T. (7^3).
197. 1024 cu. ft. hay to T. (8^3).
198. 3 T. hay to cu. ft. (7^3).
199. 3 T. hay to cu. ft. (8^3).
200. 21 cu. ft. corn in ear to bu. shelled.
201. 28 cu. ft. corn in ear to bu. shelled.
202. 50 cu. ft. corn in ear to bu. shelled.
203. 4 bu. shelled corn to cu. ft. corn in ear.
204. 3 bu. shelled corn to cu. ft. corn in ear.
205. 16 bu. shelled corn to cu. ft. corn in ear.
206. 3 ft. $\times$ 7 ft. $\times$ 2 ft. corn in ear to bu. shelled.
207. 40 cu. ft. corn in ear to bu. shelled.
208. 4 ft. $\times$ 6 ft. $\times$ 2 ft. corn in ear to bu. shelled.
209. 75 cu. ft. corn in ear to bu. shelled.
210. 1.8 bu. shelled corn to cu. ft. corn in ear.

Reduce:

211. 4 h. to degrees.
212. 4° to h.
213. 3° to time.
214. 6 h. to arc.
215. 60° to time.
216. 360° to time.
217. 14 h. to arc.
218. 11 h. to arc.
219. 6° to time.
220. 15° to time.
221. 60° to time.
222. 4 min. to arc.

DENOMINATE NUMBERS -- METRIC SYSTEM.

The Metric system is a decimal system in which all the denominations are decimal submultiples or multiples of a unit. *Ten* of any denomination make one of the next higher; that is, the multiple is 10.

The submultiples are: Latin, *mille* (1000), *centi* (100), *deci* (10). The multiples are: Greek, *Deka* (10), *Hecto* (100), *Kilo* (1000), *Myria* (10000); the abbreviation in each case is the first letter; *small*, if Latin; *capital*, if Greek.

Submultiples.	*Multiples.*
10 milli = 1 centi.	10 (*units*) = 1 Deka.
10 centi = 1 deci.	10 Deka = 1 Hekto.
10 deci = 1 (*unit*).	10 Hekto = 1 Kilo.
	10 Kilo = 1 Myria.

Units of the Different Tables.

Name.	Unit.	Abbreviation.	How obtained.	Eng. Equiv.	Important.
Long	meter	m.	.0000001 distance equator to pole.	39.37 in.	1 Km. = $\frac{5}{8}$ mi.
Land	are	a.	10 m. × 10 m.	$\frac{1}{4\,0}$ acre nearly	
Area	square meter	sq. m.	1 m. × 1 m.	10 sq. ft. +	1 sq. Km. = .3861 sq. mi.
Weight	gram	g.	wt. 1 cu. cm. water.	15.432 gr.	1 Kg. = $2\frac{1}{5}$ lb.
Capacity	liter	l.	1 cu. dm.	.908 qt. dry 1.05 qt. liquid	1 cu. m. water weighs 1 T.
Wood	stere	s.	1 cu. m.	$\frac{1}{4}$ cord nearly	

By placing the various units in the table of submultiples, the following tables are formed. Square and cubic measure, as in English, are formed by squaring and cubing long measure.

Long Measure.	*Capacity.*	*Weight.*
10 mm. = 1 cm.	10 ml. = 1 cl.	10 mg. = 1 cg.
10 cm. = 1 dm.	10 cl. = 1 dl.	10 cg. = 1 dg.
10 dm. = 1 m.	10 dl. = 1 l.	10 dg. = 1 g.
10 m. = 1 Dm.	10 l. = 1 Dl.	10 g. = 1 Dg.
10 Dm. = 1 Hm.	10 Dl. = 1 Hl.	10 Dg. = 1 Hg.
10 Hm. = 1 Km.	10 Hl. = 1 Kl.	10 Hg. = 1 Kg.
10 Km. = 1 Mm.	10 Kl. = 1 Ml.	10 Kg. = 1 Mg.
		10 Mg. = 1 Quintal (Q.).
		10 Q. = 1 Tonneau (T.).

Land.	*Square.*	*Cubic.*
10 ma. = 1 ca.	100 sq. mm. = 1 sq. cm.	1000 cu. mm. = 1 cu. cm.
10 ca. = 1 da.	100 sq. cm. = 1 sq. dm.	1000 cu. cm. = 1 cu. dm.
10 da. = 1 a., etc.	100 sq. dm. = 1 sq. m.	1000 cu. dm. = 1 cu. m.
Wood.	100 sq. m. = 1 sq. Dm.	1000 cu. m. = 1 cu. Dm.
10 ms. = 1 cs.	100 sq. Dm. = 1 sq. Hm.	1000 cu. Dm. = 1 cu. Hm.
10 cs. = 1 ds.	100 sq. Hm. = 1 sq. Km.	1000 cu. Hm. = 1 cu. Km.
10 ds. = 1 s., etc.	100 sq. Km. = 1 sq. Mm.	1000 cu. Km. = 1 cu. Mm.

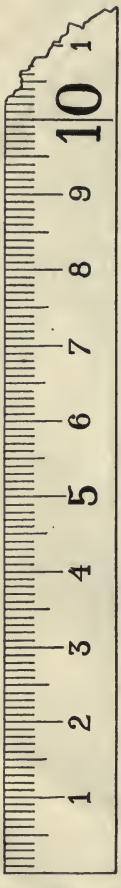
Land measure is usually given 100 ca. = 1 a.; 100 a. = 1 Ha., — the other denominations being omitted.

Wood measure is usually given 10 ds. = 1 s.

The measure of weight has two extra denominations: the quintal and tonneau.

The following denominations in italics are used in the metric system when the denominations immediately preceding them would be used in the English system. Foot, yard, rod — *meter;* mile — *kilometer;* square rod, acre — *are;* cord — *stere;* pound — *kilogram;* pint, quart, gallon, peck, bushel — *liter;* ton — *tonneau*

The metric system of weights and measures originated in France in 1795. It has been adopted in Austria, Belgium, Brazil, Denmark, Germany, Greece, Holland, Sweden, and Switzerland. Its use has been authorized by the Congress of the United States and by the Parliament of Great Britain. It is destined to displace the English system, and is already exclusively used in scientific research.



1 Decimeter of Meter Rule.

To illustrate *long measure*, prepare a strip of paper $39\frac{3}{8}$ in. long; divide it into 10 equal parts (dm.); divide each dm. into 10 equal parts (cm.); divide the first cm. into 10 equal parts (mm.).

To illustrate *land measure*, stake out on the playground a square 10 m. $\times$ 10 m.; this square will mark an *are*.

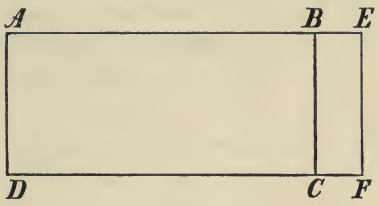
To illustrate *wood measure*, lay off on the playground a square 1 m. $\times$ 1 m.; at each corner drive a stake leaving 1 m. above ground; these stakes will hold a stere of wood.

To illustrate *weights*, procure a piece of tin-foil that weighs as much as a nickel 5¢ piece; cut the foil into five equal parts; each part will weigh 1 g. By law, the nickel must weigh 5 g. Procure a stone that weighs 2 lb. 3 oz.; it will weigh a kilogram (*nearly*).

To illustrate *measures of capacity*, draw on paper, rectangles as *ABCD* (see p. 97), having the dimensions given in the table; complete *AEFD*, and cut

it out; roll over *AD* to *BC*, and paste *BEFC*, forming cylinders; these cylinders will hold the amounts given in the table.

Name.	Length.	Breadth.
1 liter	29.16 cm.	14.78 cm.
5 deciliters	23.15 cm.	11.73 cm.
2 deciliters	17.10 cm.	8.64 cm.
1 deciliter	13.54 cm.	6.86 cm.
5 centiliters	10.74 cm.	5.44 cm.
2 centiliters	7.92 cm.	4.01 cm.
1 centiliter	6.28 cm.	3.18 cm.



1. State the unit of long measure, how it was obtained, and its English equivalent.

2. Describe in full the unit of weight (see p. 94).

3. Describe in full the unit of land measure.

4. Describe in full the unit of capacity.

5. Describe in full the unit of wood measure.

6. Name the Latin submultiples, and give their values.

7. Name the Greek submultiples, and give their values.

8. What is the multiple in all the measures except square and cubic measures? What is the multiple in square measure?

9. What is the multiple in cubic measure?

10. Give the table of multiples and submultiples.

11. Give the table of long measure. Of square measure. Of cubic measure. Of capacity.

12. Give the table of land measure in full. Give the table of land measure as abbreviated. Is the multiple 10 or 100?

13. Give the table of wood measure in full.

14. Give the table of wood measure as abbreviated.

15. Give the table of weight. What are the two extra denominations found in the table of weight?

§ 32. Practical Questions.

16. How tall are you? How long is your arm? What is the length of your forefinger? What is the thickness of your thumb-nail? What is the width of your thumb-nail?

17. What is the width of this room? What is the length of this room? What is the height of this room? What is the diameter of a nickel? What is the thickness of a nickel?

18. How far is it to the P.O.? How far is it to the nearest city? How fast does a passenger train run? How fast can a horse pace? How fast can he walk? How fast can he trot?

19. What is your weight? How much beefsteak would you buy for breakfast for three? What is the weight of a nickel? How heavy a letter of the first class will go for 2¢? What does an average horse weigh? How much hay will winter a cow? How much coal do you expect to burn in your kitchen stove this winter?

20. How much milk do you need for a cup of coffee? How much milk per day would you use for a family of six? How much will a teacup hold? How much will a tablespoon hold? What is a good yield of potatoes to the acre? What is a good yield for wheat? How large a cistern have you?

21. How much land do you need for a flower garden? What is the area of this floor? How much land is needed for a good farm?

22. What is the contents of this room in cu. m.? How much earth will make a good load for two horses? How many cu. m. are there in an ordinary rick of hay?

23. How much wood will an ordinary stove burn in winter? How much wood will produce as much heat as a ton of coal?

Note. — Give answers in the metric system.

§ 33. Reduction — Metric.

24. Tell whether Latin or Greek, and give meaning: m., D., c., M., K., d., H.

25. Give the Greek for: 100, 1000, 10, 10000.

26. Give the Latin for: 100, 1000, 10.

27. Read 102 mm.; 468 cm.; 3234 Kg.

28. Read 368 Kl.; 5600 g.; 123 s.; 160 a.; 238 Ha.

29. Read 397 Mg.; 353 cu. m.; 189 sq. mm.

30. Read 495 l.; 672 Dl.; 801 Hl.; 911 Kg.

31. Read 916 a.; 210 cl.; 717 dm.; 111 Mg.

32. Read 495 Q.; 266 cu. dm.; 889 sq. Dm.

How many:

33. mm. make 1 Dm.?

34. cm. make 1 Mm.?

35. dm. make 1 Km.?

36. m. make 1 Hm.?

37. mm. make 1 Mm.?

38. mm. make 1 dm.?

39. mm. make 1 m.?

40. mg. make 1 Mg.?

41. cg. make 1 Hg.?

42. Mg. make 1 T.?

43. Kg. make 1 T.?

44. Dg. make 1 Q.?

45. sq. mm. make 1 sq. Mm.?

46. sq. cm. make 1 sq. Dm.?

47. sq. mm. make 1 sq. dm.?

48. sq. m. make 1 sq. Hm.?

49. sq. Dm. make 1 sq. Km.?

50. cu. m. make 1 cu. Mm.?

51. cu. mm. make 1 cu. Km.?

52. cu. Dm. make 1 cu. Mm.?

53. dl. make 1 Kl.?

54. cl. make 1 Ml.?

55. Hl. make 1 Ml.?

56. a. make 1 Ha.?

Ex. **33.** 10000; mille = 1000; Deka = 10; $1000 \times 10 = 10000$.

Ex. **38.** 100; mille = 1000; deci = 10; $1000 \div 10 = 100$.

Ex. **45.** 1 with 14 ciphers; mille = 1000; Myria = 10000; 1000×10000 = 10,000,000, or 1 with 7 ciphers; 1 with 7 ciphers squared = 1 with 14 ciphers.

§ 34. Reduction — English and Metric.

Reduce approximately:

57. 16 Km. to miles.
58. 20 miles to Km.
59. 3 m. to yards.
60. 4 yards to m.
61. 600 m. to feet.
62. 84 m. to in.
63. 320 in. to m.
64. 600 feet to m.
65. 2000 sq. mi. to sq. Km.
66. 10,000 sq. Km. to sq. m.
67. 16 acres to a.
68. 400 a. to acres.
69. 24 cords to s.
70. 72 s. to cords.

Ex. **57.** 10 mi.; 1 Km. = $\frac{5}{8}$ mi.

Ex. **64.** 200 m.; 1 m. = 3 ft.; 600 ft. = 200 m.

Reduce approximately:

71. 5 gallons to l.
72. 30 l. to gallons.
73. 60 gallons to l.
74. 360 gallons to l.
75. 126 l. to gallons.
76. 1 bushel to l.
77. 256 l. to bu.
78. 64 l. to bushels.
79. 75 l. to quarts (dry).
80. 75 l. to quarts (liquid).
81. 10 l. to cu. in.
82. 25 l. to cu. in.
83. 40 l. to gallons.
84. 100 l. to gallons.

Ex. **71.** 20 l.; 1 l. = 1 qt.

Ex. **78.** 2 bu.; 1 l. = 1 qt.

Reduce approximately:

85. 10 cu. m. of water to tons.
86. 50 cu. cm. of water to grains.
87. 10 tons of water to cu. m.
88. 1 cu. dm. of water to pounds.
89. 1 Kg. to pounds.
90. 3 Hg. to pounds.
91. 1 Hg. to ounces.
92. 30 grains to g.
93. 10 g. to grains.
94. 6 tons of water to cu. m.
95. 5 bushels to l.
96. 18 Km. to miles.

PERCENTAGE.

The symbol % means *hundredths*.

Per cent and *hundredths* are interchangeable.

ILLUSTRATION.

6 % = .06
read
6 *per cent* = 6 *hundredths*.

§ 35. REDUCTION.

Change to %:

1. .07, .09, .12.
2. .16, .25, .96.
3. 325 hundredths.
4. $\frac{17}{100}$, $\frac{19}{100}$, $\frac{73}{100}$.
5. 9723 hundredths.
6. .00$\frac{1}{2}$, .00$\frac{2}{3}$, .000$\frac{1}{6}$.

Ex. 6. $\frac{1}{2}$%, etc.

Change to hundredths:

7. 17%, 13%, 14%.
8. 70%, 90%, 77%.
9. 466%, 100%, 325%.
10. $\frac{1}{2}$%, $\frac{1}{3}$%, $\frac{1}{8}$%.
11. .0$\frac{1}{4}$%, .00$\frac{1}{4}$%, .000$\frac{1}{4}$%.
12. 200%, 4000%, 7283%.

Ex. 11. .0$\frac{1}{4}$ hundredths.

Change to %:

13. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$.
14. $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$.
15. $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$.
16. $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{6}$.
17. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{8}$.
18. $\frac{5}{8}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{4}$.

Ex. 13. 50%, 33$\frac{1}{3}$%, etc.

Change to common fractions:

19. 62$\frac{1}{2}$%, 87$\frac{1}{2}$%, 37$\frac{1}{2}$%.
20. 66$\frac{2}{3}$%, 33$\frac{1}{3}$%, 16$\frac{2}{3}$%.
21. 62$\frac{1}{2}$%, 25%, 40%.
22. 75%, 60%, 25%.
23. 20%, 80%, 33$\frac{1}{3}$%.
24. 287$\frac{1}{2}$%, 233$\frac{1}{3}$%, 116$\frac{2}{3}$%.

Ex. 19. $\frac{5}{8}$, $\frac{7}{8}$, etc.

§ 36. The Operation Directly Stated.

What is:

25. 6 times 50?
26. $\frac{3}{5}$ of 50?
27. .06 of 50?
28. 6% of 50?
29. $12\frac{1}{2}$% of 40?
30. $33\frac{1}{3}$% of 60?
31. $16\frac{2}{3}$% of 36?
32. $37\frac{1}{2}$% of 16?
33. $83\frac{1}{3}$% of 24?
34. $66\frac{2}{3}$% of 72?
35. $62\frac{1}{2}$% of 32?
36. 80% of 20?
37. $87\frac{1}{2}$% of 48?
38. $87\frac{1}{2}$% of 40?
39. 75% of 144?
40. $37\frac{1}{2}$% of 64?
41. 25% of 400?
42. $133\frac{1}{3}$% of 24?
43. $266\frac{2}{3}$% of 18?
44. 325% of 160?
45. $187\frac{1}{2}$% of 40?
46. $112\frac{1}{2}$% of 64?
47. $33\frac{1}{3}$% of 16?
48. $287\frac{1}{2}$% of 16?
49. 125% of 16?
50. 25% of 34?
51. $87\frac{1}{2}$% of 17?
52. $83\frac{1}{3}$% of 25?
53. $66\frac{2}{3}$% of 49?
54. $112\frac{1}{2}$% of 33?
55. $233\frac{1}{3}$% of 12?
56. $16\frac{2}{3}$% of 36?
57. $12\frac{1}{2}$% of 80?
58. $83\frac{1}{3}$% of 30?

In these examples, since the operation is directly stated, no explanation is required.

Thus, 6 times $50 = 300$; $\frac{3}{5}$ of $50 = 30$; .06 of $50 = 3$; 6% of $50 = 3$; etc. To explain the 26th: $\frac{1}{5}$ of $50 = 10$, $\frac{3}{5}$ of $50 = 3 \times 10 = 30$, is unnecessary, because we have learned how to multiply 50 by $\frac{3}{5}$ in a previous case, and we should not at this place explain that process.

Ex. **31.** $16\frac{2}{3}$% of 36 is $\frac{1}{6}$ of 36, or 6.

Ex. **48.** $287\frac{1}{2}$% of 16 is $\frac{23}{8}$ of 16, or 46.

59. If a rope 200 ft. long shrinks 5% when wet, how long is it when wet?

Ans. 190 ft. It shrinks 5% of 200 ft., or 10 ft.; 200 ft. − 10 ft. = 190 ft.

60. A shepherd having 240 sheep, lost $16\frac{2}{3}\%$ of them in a storm; how many had he left?

61. A had \$200 and gave 40% of his money to B; how much did A retain?

62. A mine produces 2000 tons of ore; 25% of the ore is metal; 2% of the metal is silver; how many pounds of silver does it produce?

63. From a hogshead containing 480 lb. of sugar, $66\frac{2}{3}\%$ was sold at one time; 50% of the remainder, at another time; how many pounds remained?

64. A dry article weighing 60 lb. gains 10% in weight when soaked in water; how much water does it absorb?

65. The population of a town in 1890 was $16\frac{2}{3}\%$ more than 1500; what was the population?

66. If gunpowder contains 75% of saltpetre, 10% of sulphur, and 15% of charcoal, how much of each is there in 40 lb. of powder?

67. Of a regiment of 1000 men, 2% are killed, 7% are prisoners; how many are left in the regiment?

68. Of 70 children in a school $14\frac{2}{7}\%$ are boys; how many girls are there in the school?

69. A man who worked for \$24 a week had his salary diminished by $12\frac{1}{2}\%$; what was it after the deduction?

70. A farmer raising 300 bu. of wheat, sold $66\frac{2}{3}\%$ of it, and fed the rest to his stock; how much did he feed?

71. A man was hired to work 80 days, but he lost 20% of his time; how many days did he work?

§ 37. Operation to be Determined.

72. 12 is how many times 6?
73. 12 is what part of 24?
74. 12 is how many *fifths* of 24?
75. 12 is how many *hundredths* of 24?
76. 12 is what % of 24?
77. 15 is how many times 5?
78. 15 is what part of 45?
79. 15 is how many *sevenths* of 45?
80. 15 is how many *hundredths* of 45?
81. 15 is what % of 45?
82. 18 is how many *sixths* of 72?
83. 18 is how many *hundredths* of 72?
84. 18 is what % of 72?
85. 4 is what % of 2?
86. 12 is 6 times what number?
87. 12 is $\frac{2}{3}$ of what?
88. 12 is .06 of what?
89. 12 is 6% of what?
90. 15 is 3 times what?
91. 15 is $\frac{3}{5}$ of what?
92. 15 is .03 of what?
93. 15 is 3% of what?
94. 24 is $\frac{8}{9}$ of what?
95. 24 is .06 of what?
96. 24 is 6% of what?

Ex. **72.** Since 12 is some number times 6, the number is $12 \div 6$, or 2.

Ex. **73.** Since 12 is some number times 24, the number is $12 \div 24$, or $\frac{1}{2}$.

Ex. **74.** Since 12 is some number times 24, the number is $12 \div 24$, or $\frac{1}{2}$; $\frac{1}{2} = 2\frac{1}{2}$ *fifths*.

Ex. **75.** Since 12 is some number times 24, the number is $12 \div 24$, or $\frac{1}{2}$; $\frac{1}{2} = .50$.

Ex. **76.** Since 12 is some number times 24, the number is $12 \div 24$, or $\frac{1}{2}$; $\frac{1}{2} = 50\%$.

Ex. **86.** Since 12 is 6 times some number, the number is $12 \div 6$, or 2.

Ex. **87.** Since 12 is $\frac{2}{3}$ times some number, the number is $12 \div \frac{2}{3}$, or 18.

Ex. **88.** Since 12 is .06 times some number, the number is $12 \div .06$, or 200.

Ex. **89.** Since 12 is 6% times some number, the number is $12 \div .06$, or 200.

Note. — In these examples, the operation must be determined by reasoning. Each admits of being placed in the form of an equation, and the operation at once appears. Thus, Ex. 72, 12 = some no. × 6. Therefore, the no. = $12 \div 6$, since the product divided by either factor equals the other.

97. A boy having 10¢, lost 5¢; what % of his money did he lose?

Ans. 50%. Since 5¢ is some number of times 10¢, the number is 5¢ ÷ 10¢, or 50%.

98. A spent 60% of his money for a horse, 25% for a saddle, and had $30 left; what did he pay for the horse?

Ans. $120. He spent 85% of his money and had 15% left; since 15% of his money is $30, his money must be $30 ÷ .15, or $200; 60% of $200 = $120.

99. Mary had 12¢ and gave 3¢ to Henry; what % of her money remained?

100. From a cask containing 96 gallons of oil, 32 gallons were drawn; what % of the whole remained in the cask?

101. A teacher whose salary is $2400, spends $2000 annually; what % of his salary does he save?

102. The standard of gold and silver coin in the U. S. is 9 parts pure gold or silver and 1 part alloy; what % is alloy?

103. The population of a town in 1892 was 1600; in 1893 it was 2400; what was the % of increase?

104. A clerk spends $1200 a year, or $66\frac{2}{3}$% of his salary; what is his salary?

105. A man drew from the bank $575, or 25% of his deposit; what was his deposit?

106. A regiment of 800 men lost 160 men in battle; what % of the regiment remained?

107. After a battle 80% of the regiment, or 640 men, were left; how many men were there in the regiment at first?

108. A spent 60% of his money for a horse, 25% for a carriage, and had $60 left; how much did he pay for the horse? For the carriage?

109. The master of a ship threw overboard 800 bbl. of flour, or $16\frac{2}{3}$% of its cargo; what was its cargo at first?

110. What number increased by 5 times itself becomes 30?

111. What number increased by $\frac{2}{3}$ of itself becomes 30?

112. What number increased by .06 of itself becomes 212?

113. What number increased by 6% of itself becomes 212?

114. What number increased by 7 times itself becomes 40?

115. What number increased by $\frac{3}{5}$ of itself becomes 24?

116. What number increased by $16\frac{2}{3}\%$ of itself becomes 42?

117. What number increased by $83\frac{1}{3}\%$ of itself becomes 22?

118. What number diminished by $\frac{2}{5}$ of itself becomes 30?

119. What number diminished by .06 of itself becomes 188?

120. What number diminished by 6% of itself becomes 188?

121. What number diminished by $16\frac{2}{3}\%$ of itself becomes 40?

122. What number diminished by $33\frac{1}{3}\%$ of itself becomes 20?

123. What number diminished by $66\frac{2}{3}\%$ of itself becomes 20?

124. What number diminished by $87\frac{1}{2}\%$ of itself becomes 20?

125. What number diminished by 8% of itself becomes 184?

Ex. **110**. A number increased by 5 times itself becomes 6 times itself; since 6 times the number is 30, the number is $30 \div 6$, or 5.

Ex. **111**. A number increased by $\frac{2}{3}$ of itself becomes $\frac{5}{3}$ times itself; since $\frac{5}{3}$ times the number is 30, the number is $30 \div \frac{5}{3}$, or 18.

Ex. **113**. A number increased by 6% of itself becomes 106% times itself; since 106% times the number is 212, the number is $212 \div 1.06$, or 200.

Ex. **120**. A number diminished by 6% of itself becomes 94% times itself; since 94% times the number is 188, the number is $188 \div .94$, or 200.

In these examples, what the % is *of* is not given; it is therefore necessary to assume something as a base. As in the former case, the equation enables us to determine what operation to perform. Thus, Ex. 114, $8 \times \text{no.} = 40$; the no. $= 40 \div 8$.

126. A merchant sells calico for 10% more this year than last year; this year he sells for 11¢ a yard; what was last year's price?

Ans. 10¢. This year's price is 110% of last year's; since 110% of last year's price is 11¢, last year's price is 11¢ ÷ 1.10, or 10¢.

127. A merchant sells calico for 10% less this year than last year; this year he sells for 9¢ a yard; what was last year's price?

Ans. 10¢. This year's price is 90% of last year's price; since 90% of last year's price is 9¢, last year's price is 9¢ ÷ .90, or 10¢.

128. By running 15% faster than usual, a locomotive runs 690 miles a day; what is the usual distance it runs per day?

129. A field having increased in productiveness 22% over the preceding year, yielded 488 bushels of potatoes; what was the yield the previous year?

130. A farmer sold 1800 pounds of wool, which was $12\frac{1}{2}\%$ more than he sold the previous year; how many pounds did he sell the previous year?

131. A man sold 20 cows for $400, which was 20% less than they cost; what did they cost?

132. After paying 30% of his debts, a merchant found that $210 would pay the remainder; what did he owe at first?

133. My salary this year is $75 per month, or 25% more than last year; what was my salary last year?

134. A dealer has two kinds of apples: the first kind he sells for $66\frac{2}{3}\%$ more than the second; he sells the first for $1.50 a barrel; for how much does he sell the second kind?

135. A sells a cow for $49 which is $12\frac{1}{2}\%$ less than he gets for his horse; what does he get for the horse?

136. I pay $12 per week for board this year, which is 20% less than I paid last year; what did I pay last year?

Names are sometimes given to the terms used in percentage.

What the % is of, is the *base;* the base × the %, the *percentage;* base + percentage, *amount;* base − percentage, *difference;* the %, the *rate.*

ILLUSTRATION.
6 % of 200 = 12.
200 + 12 = 212.
200 − 12 = 188.
200, *base.*
12, *percentage.*
212, *amount.*
188, *difference.*
6 %, *rate* %.

To find percentage:

137. Base 60; rate 7%.
138. Amount 80; base 60.
139. Difference 70; base 90.
140. Difference 40; rate 20%.
141. Amount 63; rate $12\frac{1}{2}$%.

To find amount:

142. Base 60; rate $16\frac{2}{3}$%.
143. Percentage 30; base 60.
144. Percentage 25; rate 5%.
145. Percentage 400; base 4000.

To find difference:

146. Base 32; rate 25%.
147. Percentage 24; base 30.
148. Base 72; rate $33\frac{1}{3}$%.
149. Percentage 25; rate 10%.
150. Percentage 30; rate 15%.

To find rate %:

151. Base 60; percentage 40.
152. Amount 70; percentage 30.
153. Difference 60; percentage, 20.
154. Amount 75; base 60.
155. Difference 70; base 80.

To find base:

156. Rate 6%; percentage 12.
157. Rate 6%; amount 212.
158. Rate 6%; difference 188.
159. Amount 90; percentage 10.
160. Rate 8%; percentage 80.

To find base:

161. Rate 8%; amount 324.
162. Rate 8%; difference 368.
163. Difference 70; percentage 30.
164. Rate 10%; percentage 70.

§ 38. Profit and Loss.

What is the selling price?

165. Cost \$10; gain \$2.
166. Cost \$36; gain $83\frac{1}{3}\%$.
167. Cost \$24; gain $12\frac{1}{2}\%$.
168. Cost \$25; gain 40%.
169. Cost \$54; gain $33\frac{1}{3}\%$.
170. Cost \$50; loss \$5.
171. Cost \$12; loss 15%.
172. Cost \$20; loss $66\frac{2}{3}\%$.
173. Loss \$16; loss 8%.
174. Gain \$ 9; gain 75%.

What is the cost?

175. Selling price $4¢$; gain $33\frac{1}{3}\%$.
176. Selling price \$21; gain \$6.
177. Selling price \$30; gain $87\frac{1}{2}\%$.
178. Selling price \$20; gain 25%.
179. Selling price \$1.90; loss 5%.
180. Selling price \$25; loss \$5.
181. Selling price $4¢$; loss $66\frac{2}{3}\%$.
182. Loss $5¢$; loss $83\frac{1}{3}\%$.
183. Gain \$9; gain 5%.

What is the gain or loss %?

184. Selling price \$12; cost \$10.
185. Selling price \$12; gain \$3.
186. Selling price \$12; loss \$3.
187. Cost \$12; gain \$3.
188. Cost \$12; loss \$3.
189. Cost \$25; selling price \$30.

The gain or loss is always regarded as some per cent of the cost.

Ex. **166.** The gain is $\frac{5}{6}$ of \$36, or \$30; the selling price is \$66.

Ex. **175.** 100% of C = cost; $33\frac{1}{3}\%$ of C = gain; $133\frac{1}{3}\%$ of C, or $\frac{4}{3}$ of cost $= 4¢$; cost $= 4¢ \div \frac{4}{3}$, or $3¢$.

Ex. **184.** The gain is \$2; \$2 = no. × \$10; no. $= 2 \div 10$, or 20%.

190. By selling a horse for $250 I gained 25%; what did the horse cost?

Ans. $200. 100% of C = cost; 25% of C = gain; 125% of C = $250; cost = $250 ÷ 1.25 = $200.

191. By selling a horse for $270 I lost 10%; what did the horse cost?

Ans. $300. 100% of C = cost; 10% of C = loss; 90% of C = $270; cost = $270 ÷ .90 = $300.

192. What must be the selling price of tea that cost 80¢ a pound, to gain 20%? 96

193. What % is gained on an article bought at $4.50 and sold for $6? $33\frac{1}{3}\%$

194. A grocer sells corn at a profit of 12¢ a bushel and thereby gains 20%; what is the cost? 60 ¢

195. Papers were sold for 5¢ each at a gain of 25%; what was the gain on 4 papers? 4¢

196. By selling books at $1.88 there was a loss of 6%; what was the cost? $200

197. Two horses were sold for $99 each; on the first there was a loss of 10%; on the second a gain of 10%; what was paid for both horses? $198 cost

198. Do I gain or lose on the sale of both horses in example 197? How much? 1 ¢ loss.

199. A merchant by selling silks for $12 more than they cost gained $66\frac{2}{3}\%$; what was the selling price? 30

200. Find the profit on land that cost $200 and was sold at a gain of 12%. 24

201. Find the selling price of grain that cost $200 and was sold at a loss of 9%.

202. Find the % gained on oil bought at 12¢ and sold at 14¢.

203. A watch that cost $100 was sold at a gain of 10%; what was the gain? What was the selling price?

204. What is gained *per cent* by selling coal at $6 a ton that cost $5 a ton?

205. A horse was sold at a loss of $50, which was 10% of the cost; what was the cost?

206. A watch was sold for $240, at a gain of 20%; what was the cost?

207. By selling a cow for $24 more than she cost, a farmer gained $37\frac{1}{2}\%$; what was the selling price?

208. What must be the selling price of tea that cost 30¢ a lb., in order to gain $33\frac{1}{3}\%$?

209. A boy, by selling newspapers at 5¢ each, gains $66\frac{2}{3}\%$; what do they cost him?

210. A boy, by selling newspapers at 5¢ each, gains 60% on the selling price; what do they cost him?

211. An article was bought for $4 and sold for $6; what was the gain per cent? What per cent of the selling price was gained?

212. How shall I mark goods that cost $12, so as to gain $16\frac{2}{3}\%$?

213. How shall I mark goods that cost $5, so that I can deduct 10% from the marked price, and yet make 8% on the cost?

214. What % is gained on goods sold at double the cost?

215. A carpet which cost $12 was sold for $9; what was the loss %?

216. How must goods that cost $15 be marked so as to gain $12\frac{1}{2}\%$?

217. How shall I mark goods that cost $6, so that I can deduct 10% from the marked price, and make 5% on the cost?

218. How must I sell goods that cost $56 so as to gain $37\frac{1}{2}\%$?

§ 39. Commission.

A person may be employed to buy or sell for another; the employer is the *principal;* the other, the *agent;* the price paid for the service, the *commission;* the amount returned to the principal, the *net proceeds.*

Illustration. — A farmer takes to a grocer 10 barrels of apples to be sold at $2 a barrel, agreeing to pay him 10% commission for selling them. It is just that the grocer should keep 10% of the entire sales. Hence,

If an agent sells, his commission is some per cent of the sales.

Illustration. — A woolen manufacturer sends his agent $1050 with instructions to buy wool after deducting his commission of 5%. If the agent took 5% of $1050, he would take 5% of what he pays for the wool, and also 5% of what he keeps. It is not just that he should receive 5% of what he keeps, because he performs no labor for it, but he is entitled to 5% of what he pays for wool. Hence,

If an agent buys, his commission is some per cent of the purchase.

219. Find the amount of sales when the principal receives $40, at a commission of 20%.

Ans. $50. The commission is 20% of the sales; 100% of sales = the sales; the sales − the commission = the net proceeds; 80% of sales = $40; sales = $40 ÷ .80 = $50. *Proof.* 20% of $50 = $10; $50 − $10 = $40.

220. An agent sells 10 bbl. of apples at $2 a bbl.; what is his commission at 10%.

Ans. $2. His commission is 10% of the selling price, or 10% of $20, or $2.

221. Find the rate of commission when $2 is paid for a sale of $8.

Ans. 25%. The commission is some per cent of the sales; if $2 is some number times $8, the number is 2 ÷ 8, or 25%. *Proof.* 25% of $8 = $2.

222. A woolen manufacturer sends his agent \$1050 to invest in wool after deducting 5% commission; what is the purchase price and what is the commission?

Ans. \$1000; \$50. 5% of purchase is commission; 100% of purchase is the purchase; 105% of purchase = \$1050; purchase is \$1050 ÷ 1.05, or \$1000; the commission is 5% of \$1000, or \$50. *Proof.* 5% of \$1000 = \$50; \$1000 + \$50 = \$1050.

223. Find the amount of sales when an agent receives \$4 from a 2% commission.

Ans. \$200. The commission is some per cent of the sales; 2% of the sales is \$4; the sales are \$4 ÷ .02, or \$200. *Proof.* 2% of \$200 = \$4.

224. Find the commission on the sale of a farm for \$1000, at 3%.

225. Find the commission on the purchase of a mill for \$1000, at 3%.

226. Find the commission when an agent receives \$220 to be invested in goods after deducting his commission of 10%.

227. How many pounds of sugar, at 8¢ a pound, can an agent buy for \$40.80 after deducting his commission of 2%?

228. Find the rate of commission when \$2 is paid for a sale of \$10.

229. Find the amount of sales when a commission of 2% pays the agent \$8.

230. Find the commission on the sale of a house for \$20,000, at 5%.

231. Find the rate of commission when an agent receives \$5 for a sale of \$200.

232. Find the commission when an agent receives \$3360 to be invested in goods after deducting his commission of 12%.

233. Find the amount of sales when a commission of 6% pays the agent \$600.

234. Find the net proceeds from the sale of 20 barrels of sugar at \$4, commission 10%.

235. Find the amount of the purchase, when an agent invests \$440 in sugar after deducting his commission of 10%.

236. A lawyer, having a debt of \$7000 to collect, settles for 60%; his commission is $1\frac{1}{4}\%$; how much does he remit?

237. A lawyer collects a debt, takes 2% for his fee, and remits the balance, or \$490; what is his fee?

238. An agent receives \$5050 with which to purchase goods, after deducting his commission of 1%; what was the cost of the goods?

239. A buys corn at $1\frac{1}{2}\%$ commission, and $2\frac{1}{2}\%$ for guaranteeing payment; if the whole cost, including commission and guaranty is \$416, what was the first cost of the corn?

240. My agent in Paris has bought for me 16 bales of calico, each bale containing 50 pieces of 30 meters each, at 20¢ a meter; his commission is 1%; how much must I send him?

241. In buying shoes at a commission of $2\frac{1}{2}\%$, an agent's commission was \$25; how much did he invest?

242. An agent sells 10000 lb. of sugar at 8¢ per pound, and receives $1\frac{1}{2}\%$ commission; he pays \$10 for freight; find his commission and the net proceeds.

243. An agent receives \$1010 with which to buy shoes and pay his commission of 1%; what does he pay for the shoes? What is his commission?

244. D bought a horse for \$250, paying 2% of the cost for commission, and 2% of cost for traveling expenses; he sold him at an advance of 10% on the entire cost, including commission and expenses; how much did he gain?

INTEREST.

Money paid for the use of money is *interest;* the money loaned is the *principal;* the sum of the principal and interest is the *amount.*

There are three conceptions of interest:

That the principal alone bears interest, *simple interest.*

That the principal and the interest on the principal at the end of each year bear interest, *annual interest.*

That the principal and the interest on the principal at the end of each year, and all other unpaid interest at the end of each year bear interest, *compound interest.*

Unless otherwise stated, simple interest is always understood. The rate of interest is regulated by law; whenever interest is in excess of the legal rate, it is called *usury.*

If $6 is paid for the use of $100, $100 is the *principal;* $6, *interest;* $106, *amount.*

Suppose $100 is loaned for 3 yr., at 6%, and the interest remains unpaid until the end of this period.

At the end of the first year, $6 interest is due by each of the three conceptions.

At the end of the second year, by the first conception, another $6, or $12 in all, is due; by each of the second and third, in addition, the interest of the first $6 for 1 yr. (36¢), or $12.36 in all.

At the end of the third year, by the first conception, another $6 is due, or $18 in all; by the second, in addition, the interest of the first $6 for 2 yr. (72¢), and the interest of the second $6 for 1 yr. (36¢), or $19.08 in all; by the third, in addition, the interest of the first 36¢ for 1 yr. (2.16¢), or $19.1016 in all.

For 1 yr. the simple, annual, and compound interest are the same; for 2 yr., the annual and compound interest are the same, and greater than the simple; for 3 yr. or more, each is different, the order of magnitude being compound, annual, simple.

§ 40. Simple Interest.

1. What is the interest of $1 for 1 yr. at 6%?

2. What is the interest of $1 for 1 mo. at 6%?

3. What is the interest of $1 for 1 da. at 6%?

The interest of $1 for 1 yr. at 6% is .06 of $1, or 6¢.

Since the interest of $1 for 12 mo. is 6¢, for 1 mo. it is $\frac{1}{12}$ of 6¢, or 5 m.

Since the interest of $1 for 30 da. is 5 m., for 1 da. it is $\frac{1}{30}$ of 5 m., or $\frac{1}{6}$ of a m.

To be memorized.

The interest of $1 for 1 yr. at 6% is 6¢; for 1 mo., $\frac{1}{2}$¢; for 1 da., $\frac{1}{6}$ of a mill.

What is the interest of:

4. $1 for 2 yr. at 6%?
5. $2 for 3 yr. at 6%?
6. $4 for 5 yr. at 6%?
7. $1 for 2 mo. at 6%?
8. $1 for 3 mo. at 6%?
9. $1 for 5 mo. at 6%?
10. $2 for 7 mo. at 6%?
11. $3 for 9 mo. at 6%?
12. $5 for 7 mo. at 6%?
13. $1 for 4 da. at 6%?
14. $1 for 10 da. at 6%?
15. $3 for 6 da. at 6%?
16. $4 for 8 da. at 6%?
17. $1 for 2 yr. 2 mo. 2 da. at 6%?
18. $10 for 3 yr. 18 da. at 8%?
19. $30 for 4 yr. 8 mo. at 6%?
20. $100 for 63 da. at 10%?
21. $1 for 5 yr. 5 mo. 5 da. at 10%?
22. $1 for 33 da. at 12%?

Ex. **18.** $2.44. The interest of $1 for 3 yr. at 6% is $.18; for 18 da., $.003; for the whole time, $.183; of $10, 10 times $.183, or $1.83; at 8%, $\frac{1}{3}$ more, or $2.44.

Ex. **20.** $1.75. The interest of $1 for 63 da. at 6% is $.0105; of $100, $1.05; at 10%, $\frac{2}{3}$ more, or $1.75.

What is the interest of $1:

23. For 3 yr. at 4%?
24. For 5 yr. at 7%?
25. For 8 yr. at 9%?
26. For 2 yr. at $5\frac{1}{2}$%?
27. For 7 yr. at 2%?
28. For 5 mo. at 4%?
29. For 7 mo. at 7%?
30. For 9 mo. at 4%?
31. For 4 mo. at 9%?
32. For 8 mo. at 12%?
33. For 1 da. at 7%?
34. For 1 da. at 8%?
35. For 3 da. at 9%?
36. For 7 da. at 12%?
37. For 5 da. at 2%?
38. For 2 yr. 5 mo. at 7%?
39. For 7 yr. 7 mo. at 8%?
40. For 8 yr. 5 mo. at 6%?

What is the amount of $1:

41. For 2 yr. 6 mo. at 8%?
42. For 3 yr. 4 mo. at 6%?
43. For 5 yr. 8 mo. 6 da. at 6%?
44. For 1 yr. 10 mo. 24 da. at 5%?
45. For 9 mo. 18 da. at 9%?
46. For 3 mo. 12 da. at 6%?
47. For 7 yr. at 7%?
48. For 4 yr. 10 mo. at 6%?
49. For 8 mo. 12 da. at 3%?
50. For 7 mo. 6 da. at 4%?
51. For 6 mo. 9 da. at 6%?
52. For 4 mo. 24 da. at 8%?
53. For 1 yr. 6 mo. at 7%?
54. For 5 yr. 9 mo. at 6%?
55. For 10 yr. 1 mo. 6 da. at 2%?
56. For 12 yr. 10 mo. 3 da. at 6%?

Ex. **23**. 12¢. When years alone, months alone, or days alone are given, it may be simpler to find the interest directly without the 6% method. Thus, the interest of $1 for 1 yr. at 4% is 4¢, for 3 yr. 12¢.

Ex. **41**. $1.20. The amount is the interest plus the principal.

What principal will gain:

57. $24 in 4 yr. at 6%?
58. $36 in 3 yr. at 2%?
59. $48 in 6 yr. at 4%?
60. $56 in 2 yr. at 7%?
61. $63 in 3 yr. at 3%?
62. $72 in 9 yr. at 4%?
63. $20 in 2 yr. at 5%?
64. $30 in 3 mo. at 6%?
65. $32 in 4 mo. 24 da. at 8%?
66. $330 in 6 mo. 18 da. at 6%?
67. $100 in 5 mo. 30 da. at 5%?
68. $120 in 10 mo. at 12%?
69. $40 in 4 mo. at 6%?
70. $60 in 3 yr. at 5%?
71. $15 in 9 mo. at 10%?
72. $30 in 10 mo. at 6%?
73. $80 in 1 yr. 4 mo. at 12%?
74. $50 in 2 yr. 6 mo. at 10%?

What principal will amount to:

75. $496 in 4 yr. at 6%?
76. $236 in 2 yr. at 9%?
77. $600 in 2 yr. at 10%?
78. $620 in 3 yr. at 8%?
79. $226 in 2 yr. 2 mo. at 6%?
80. $318 in 8 mo. at 9%?
81. $540 in 3 yr. 6 mo. at 10%?
82. $256 in 4 yr. at 7%?
83. $345 in 3 yr. at 5%?
84. $396 in 4 yr. at 8%?
85. $516 in 8 yr. at 9%?
86. $412 in 4 mo. at 9%?
87. $205 in 5 mo. at 6%?
88. $254 in 9 yr. at 3%?
89. $312 in 6 mo. at 8%?
90. $416 in 4 mo. at 12%?
91. $321 in 7 mo. at 12%?
92. $436 in 18 mo. at 6%?

Ex. **57.** $100. Assume $1. $1 in 4 yr. at 6% will gain 24¢; it will take as many dollars to gain $24 as 24¢ is contained times in $24, or $100.

Ex. **75.** $400. Assume $1. $1 in 4 yr. at 6% will amount to $1.24; it will take as many dollars to amount to $496, as $1.24 is contained times in $496, or $400.

In what time will:

93. \$100 gain \$24 at 6%?
94. \$200 gain \$30 at 5%?
95. \$150 gain \$36 at 4%?
96. \$300 gain \$42 at 7%?
97. \$400 gain \$64 at 8%?
98. \$108 gain \$27 at 5%?
99. \$125 gain \$20 at 2%?
100. \$100 gain \$90 at 9%?
101. \$250 gain \$50 at 10%?
102. \$425 gain \$51 at 12%?
103. \$325 gain \$39 at 3%?
104. \$500 gain \$60 at 6%?
105. A sum gain $\frac{1}{2}$ of itself at 6%?
106. A sum gain $\frac{1}{3}$ of itself at 8%?
107. A sum gain $\frac{2}{5}$ of itself at 10%?
108. A sum gain itself at 8%?
109. A sum gain $\frac{1}{4}$ of itself at 12%?
110. A sum gain 3 times itself at 4%?
111. In what time will \$100 amount to \$124 at 6%?
112. \$50 to \$52 at 4%?
113. \$25 to \$40 at 6%?
114. \$40 to \$68 at 7%?
115. \$40 to \$72 at 8%?
116. \$80 to \$84 at 5%?
117. \$70 to \$84 at 2%?
118. \$30 to \$33 at 10%?
119. \$40 to \$88 at 12%?
120. \$50 to \$75 at 5%?
121. \$50 to \$53 at 6%?
122. \$30 to \$36 at 10%?
123. \$40 to \$76 at 9%?
124. \$50 to \$62 at 6%?
125. \$60 to \$84 at 4%?
126. In what time will a sum amount to twice itself at 6%?
127. In what time will a sum quadruple at 10%?
128. In what time will a sum double at 7%?

Ex. **93.** 4 yr. Assume 1 yr. \$100 in 1 yr. at 6% will gain \$6; it will take as many years to gain \$24 as \$6 is contained times in \$24, or 4 years.

Ex. **105.** 8 yr. 4 mo. This means, 'in what time will \$12 (any principal) gain \$6 at 6%?' Assume 1 yr., etc.

Ex. **111.** 4 yr. This means, 'in what time will \$100 gain \$24 at 6%?' Assume 1 yr., etc.

Ex. **127.** 30 yr. This means, 'in what time will \$10 (any principal) gain \$30 at 10%?' Assume 1 yr., etc.

At what % will:

129. $100 gain $24 in 4 yr.?
130. $200 gain $98 in 7 yr.?
131. $300 gain $81 in 9 yr.?
132. $500 gain $75 in 5 yr.?
133. $400 gain $84 in 3 yr.?
134. $100 gain $36 in 6 yr.?
135. $175 gain $70 in 2 yr.?
136. $800 gain $96 in 8 yr.?
137. $600 gain $60 in 5 yr.?
138. $190 gain $76 in 4 yr.?
139. $250 gain $35 in 7 yr.?
140. $100 gain $81 in 9 yr.?

141. A sum gain $\frac{1}{2}$ of itself in 8 yr.?
142. A sum gain $\frac{1}{3}$ of itself in 6 yr.?
143. A sum gain $\frac{2}{5}$ of itself in 7 yr.?
144. A sum gain itself in 8 yr.?
145. A sum gain two times itself in 10 yr.?
146. A sum gain three times itself in 5 yr.?

147. At what % will $100 amount to $124 in 4 yr.?

148. $40 to $72 in 8 yr.?
149. $80 to $88 in 5 yr.?
150. $20 to $38 in 9 yr.?
151. $20 to $48 in 7 yr.?
152. $80 to $96 in 2 yr.?
153. $60 to $96 in 3 yr.?
154. $40 to $84 in 10 yr.?
155. $30 to $39 in 6 yr.?
156. $20 to $42 in 11 yr.?
157. $30 to $48 in 12 yr.?
158. $20 to $27 in 7 yr.?
159. $20 to $29 in 9 yr.?

160. At what % will a sum amount to twice itself in 10 yr.?
161. At what % will a sum triple in 20 yr.?
162. At what % will a sum double in 10 yr.?
163. At what % will a sum quadruple in 12 yr.?

Ex. **129.** 6%. Assume 1%. $100 in 4 yr. at 1% will gain $4; it will take as many % to gain $24 as $4 is contained times in $24, or 6%.

Ex. **141.** $6\frac{1}{4}$%. This means, 'at what % will $12 (any principal) gain $6 in 8 yr.?' Assume 1%, etc.

Ex. **147.** This means, 'at what % will $100 gain $24 in 4 yr.?' Assume 1%, etc.

Ex. **161.** 10%. This means, 'at what % will $12 (any principal) gain $24 in 20 yr.?' Assume 1%, etc.

164. Is it proper to reason thus: "Since \$1 amounts to \$1.06, \$5 will amount to 5 times \$1.06, or \$5.30"? Why?

Yes. Because \$5 will amount to 5 times as much as \$1.

165. Is it proper to reason thus: "Since \$1 amounts to \$1.06 in 1 yr., in 2 yr. it will amount to 2 times \$1.06, or \$2.12"? Why?

No. The amount is in every case *once* the principal plus the interest. By reasoning as at the left, we make the amount *twice* the principal plus the interest.

166. If \$60 amounts to \$70 in 1 yr., what will it amount to in 2 yr.?

167. If \$40 amounts to \$70 in 5 yr., what will it amount to in 10 yr.?

168. If \$60 amounts to \$100 in 4 yr., what will it amount to in 8 yr.?

169. At what % will \$200 gain \$56 in 4 yr.?

170. At what % will a sum triple itself in 40 yr.?

171. At what % will a sum gain 3 times itself in 30 yr.?

172. What principal will gain \$200 in 3 yr. at 10%?

173. What principal will amount to \$224 in 2 yr. at 6%?

174. In what time will \$200 gain \$160 at 8%?

175. In what time will \$200 amount to \$256 at 7%?

176. In what time will a sum gain 3 times itself at 10%?

177. In what time will a sum quadruple itself at 2%?

178. What is the interest of \$100 for 2 yr. 6 mo. at 8%?

179. How many dollars in 6 yr. at 3% will gain the interest of \$100 for 4 yr. at 6%?

180. If \$30 amounts to \$60 in 3 yr., what is the rate of interest? The amount in 4 yr.? The interest each year?

When the time does not exceed 123 days, a modification of the 6% method is in general use.

Moving the decimal point two places to the left in the principal, gives the interest for 60 days at 6%.

Since the interest of $1 for 60 da. at 6% is 1¢, and 1¢ is .01 of $1, dividing the principal by 100, that is, moving the decimal point two places to the left, will give the interest for 60 days at 6%.

What is the interest of:

181. $125 for 60 da. at 6%?
182. $250 for 60 da. at 6%?
183. $313 for 60 da. at 6%?
184. $400 for 63 da. at 6%?
185. $100 for 93 da. at 6%?
186. $200 for 33 da. at 6%?
187. $100 for 63 da. at 10%?
188. $100 for 93 da. at 10%?
189. $100 for 33 da. at 10%?
190. $100 for 63 da. at 8%?
191. $100 for 93 da. at 8%?
192. $100 for 33 da. at 8%?
193. $100 for 63 da. at 12%?
194. $100 for 33 da. at 12%?
195. $100 for 93 da. at 12%?
196. $200 for 93 da. at 7%?
197. $100 for 63 da. at 7%?
198. $200 for 33 da. at 7%?
199. $500 for 63 da. at 6%?
200. $800 for 33 da. at 6%?
201. $200 for 93 da. at 6%?
202. $300 for 33 da. at 10%?
203. $200 for 93 da. at 10%?
204. $400 for 63 da. at 10%?
205. $400 for 33 da. at 10%?
206. $800 for 63 da. at 8%?

Ex. **184.** $4.20. $4 (moving the decimal point two places to the left) is the interest for 60 days; $\frac{1}{2}\frac{1}{0}$ of $4, or 20¢, the interest for 3 days.

Ex. **191.** $2.07. $1 is the interest for 60 days at 6%; $\frac{1}{2}$ of $1, or 50¢, the interest for 30 days; $\frac{1}{1}\frac{1}{0}$ of 50¢, or 5¢, the interest for 3 days, $1.55, the interest at 6%; $1.55 + $\frac{1}{3}$ of $1.55, or $2.07, the interest at 8%.

Ex. **202.** $2.75. $3 is the interest for 60 days at 6%; $1.50, for 30 days; 15¢ for 3 days; $1.65, the interest at 6%; 10% is $\frac{1}{6}\frac{0}{0}$ of 6%; $1.65 × 10 × $\frac{1}{6}$ = $2.75, the interest at 10%. The last step is taken by moving the decimal point one place to the right, and dividing by 6.

§ 41. Trade Discount.

Merchants and manufacturers usually have *fixed price lists* of their goods, and when the market varies they change the *rate of discount* instead of changing the fixed price. The fixed price is the *list price;* the deduction is the *trade discount*, the amount paid, the *net price.*

They announce their terms upon their bill heads thus, "Terms 30 days less 5%," etc. In addition to this, they frequently offer an additional discount for cash.

Sold a bill of goods, list price $20, on 4 mo. at 5% discount, and deducted 10% for cash; what was the net price?

Ans. $17.10. This means that payment was not due for 4 mos.; that 5% of list price was to be deducted because of the condition of the market; and that an additional discount of 10%, after the first discount had been subtracted, was made for cash.

$20 less 5% of $20 = $19; $19 less 10% of $19 = $17.10.

207. Sold a bill of goods, list price $20, on 3 mo. at 10% discount, and deducted 5% for cash; what was the net price?

208. Sold a bill of goods, list price $20, on 3 mo. at 5% discount, and deducted 10% for cash; what was the net price?

209. Compare examples 207 and 208. Does it make any difference with the net price whether the discount is 10% off and 5% for cash, or 5% off and 10% for cash?

210. Which is the better for the purchaser, 10% off and 5% for cash, or 5% off and 10% for cash?

211. Compare examples 207 and 208 and decide whether any account should be taken of the time before the bill is due in computing the net price.

§ 42. True Discount.

The true present worth of a *long-time* note, or of a sum of money due a long time in advance, is that sum which, put at interest now, will amount to the debt at the expiration of the time.

John Smith owes me $212 a year from to-day; what is the present worth, money at 6%? What is the discount?

Ans. Present worth $200; discount $12. The present worth is that sum which put at interest to-day will amount to $212 in 1 yr. Assume $1. $1 in 1 yr. at 6% amounts to $1.06; it will take as many dollars to amount to $212, as $1.06 is contained times in $212, or $200. The discount is the debt minus the present worth, or $12.

Find the present worth of:

212. $412 due in 6 mo., int. 6%.
213. $324 due in 8 mo., int. 12%.
214. $321 due in 1 yr., int. 7%.
215. $430 due in 1 yr. 6 mo., int. 5%.
216. $520 due in 8 mo., int. 6%.
217. $340 due in 1 yr. 4 mo., int. 10%.
218. $210 due in 10 mo., int. 6%.
219. $590 due in 2 yr., int. 9%.
220. $336 due in 3 yr., int. 4%.
221. $644 due in 5 yr., int. 8%.
222. $427 due in 9 mo., int. 9%.
223. $324 due in 1 yr. 4 mo., int. 6%.
224. $230 due in 1 yr. 8 mo., int. 9%.
225. $300 due in 4 yr. 2 mo., int. 12%.
226. $266 due in 5 yr. 6 mo., int. 6%.

Ex. **212.** $400. $1 will amount to $1.03 in 6 mo. at 6%; it will take as many dollars to amount to $412 as $1.03 is contained times in $412, or $400.

§ 43. Bank Discount.

The true method of finding the present worth of a *short-time* note is to find that sum which will amount to the face of the note in the given time at the given rate. But since the *interest* on the face for the given time at the given rate is nearly the same, more easily found, and to the advantage of the lender, the latter method, known as *Bank Discount*, is employed.

The payer is allowed, on all notes, *three days*, called *days of grace*, for payment, after the note becomes due. Hence,

In finding bank discount, three days are always added to the time.

In this set the days of grace are included.

What is the bank discount of:

227. \$360 for 33 da. at 6%?
228. \$240 for 63 da. at 6%?
229. \$360 for 33 da. at 10%?
230. \$360 for 63 da. at 10%?
231. \$430 for 33 da. at 6%?
232. \$520 for 63 da. at 9%?
233. \$640 for 93 da. at 6%?
234. \$150 for 33 da. at 6%?
235. \$260 for 63 da. at 6%?
236. \$830 for 93 da. at 6%?
237. \$240 for 93 da. at 6%?
238. \$240 for 33 da. at 12%?
239. \$240 for 63 da. at 12%?
240. \$240 for 93 da. at 12%?
241. \$160 for 63 da. at 8%?
242. \$370 for 63 da. at 4%?
243. \$280 for 33 da. at 5%?
244. \$410 for 93 da. at 6%?
245. \$190 for 63 da. at 6%?
246. \$200 for 33 da. at 6%?
247. \$900 for 33 da. at 10%?
248. \$720 for 63 da. at 8%?
249. \$650 for 93 da. at 6%?
250. \$875 for 33 da. at 4%?

Ex. **227.** \$1.98. The interest of \$360 for 60 da. at 6%, is \$3.60; for 30 da., \$1.80; for 3 da., \$.18; for 33 da., \$1.98.

§ 44. Stocks.

To raise money for the prosecution of business enterprises, stock companies are often formed. Shares are issued with a face value (*par value*) of $100, but sometimes this is made $50, or $25.

Shares do not often sell for their par value, but for more (*above par*), or for less (*below par*), according to the success of the enterprise.

Earnings (*dividends*) are paid at certain periods.

Brokers buy and sell stock for their principals, charging something (*brokerage*) both for buying and selling.

Illustration.

On a mining claim in Mexico, gold and silver were found in such abundance that a stock company was organized. They issued 100000 shares with a face value of $100 a share, but sold them all for $2 a share. The earnings at the end of the first year were $660000, and a dividend of 6% was declared. After the dividend, the stock sold for $110 a share. At this time James Lyman bought of John Fluker, through a broker, 50 shares, paying $\frac{1}{8}$% brokerage, and received the certificate represented. He afterwards sold the shares at $75, paying $\frac{1}{4}$% brokerage.

On this certificate \$100 is the *par value* of each share; \$2, \$110, \$75 were the *market values* at different times.

The par value is \$100 unless otherwise stated.

The brokerage and dividend are some % of the par value.

This understanding saves confusion, and makes it unnecessary to call attention to the par value.

The brokerage and dividend must be some per cent either of the *market* value or of the *par* value. The par value never changes, the market value is constantly changing; hence the former is selected.

251. What was the par value of the 50 shares when bought by John Fluker? What was the market value?

Ans. Par value, \$5000; market value, \$100.

252. What was the par value of the 50 shares when bought by James Lyman? What was the market value?

Ans. Par value, \$5000; market value, \$5500.

253. How much stock did John Fluker own?

Ans. \$5000 stock.

254. How much stock did James Lyman own?

Ans. \$5000 stock.

255. What was the income on one share at the time of first dividend?

Ans. \$6. The income on one share was 6% of \$100, or \$6.

256. How much brokerage did Lyman pay on one share when he bought? When he sold?

Ans. \$$\frac{1}{8}$ when he bought, \$$\frac{1}{4}$ when he sold. The brokerage was $\frac{1}{8}$%, and $\frac{1}{4}$% of \$100.

257. How much did James Lyman pay per share? How much did he receive?

Ans. He paid \$$110\frac{1}{8}$. \$110+\$$\frac{1}{8}$ brokerage. He received \$$74\frac{3}{4}$. \$75 −\$$\frac{1}{4}$ brokerage.

258. Does the market value affect the dividend?

Ans. No. The dividend is some % of the *par value*.

259. Does the dividend or the market value in any way affect the brokerage?

Ans. No. The brokerage is some % of the *par value*.

260. What is the dividend on one share of 6% stock bought at 50 and sold at 60?

Ans. $6. The dividend is some % of the par value.

261. What is the market value of $6000 5% stock?

Ans. We have no means of knowing.

How many shares in:

262. $500 5% stock?
263. $8000 3% stock?
264. $1000 7% stock?
265. $1200 10% stock?
266. $1100 6% stock?
267. $15000 8% stock?
268. $2000 9% stock?
269. $1400 12% stock?
270. $9900 3% stock?
271. $2500 4% stock?

Find the cost of:

272. $800 stock, at 80, brokerage $\frac{1}{8}$?
273. $400 stock, at 90, brokerage $\frac{1}{4}$?
274. $500 stock, at 60, brokerage $\frac{1}{5}$?
275. $1200 stock, at 50, brokerage $\frac{1}{4}$?
276. $1000 stock, at 70, brokerage $\frac{1}{2}$?
277. $8100 stock, at 80, brokerage $\frac{1}{9}$?
278. $7200 stock, at 50, brokerage $\frac{1}{8}$?
279. $400 stock, at 40, brokerage $\frac{3}{4}$?
280. $600 stock, at 90, brokerage $\frac{1}{3}$?
281. $700 stock, at 60, brokerage $\frac{1}{7}$?

Ex. **262.** 5 shares. The par value of one share is $100; $500 is the par value of as many shares as $100 is contained times in $500, or 5 shares.

Ex. **272.** $641. The entire cost of one share is 80\frac{1}{8}$; the cost of 8 shares is 8 times 80\frac{1}{8}$, or $641.

Find the net proceeds of:

282. \$800 stock sold at $80\frac{1}{8}$, brokerage $\frac{1}{8}$.
283. \$400 stock sold at 90, brokerage $\frac{1}{4}$.
284. \$500 stock sold at $50\frac{1}{4}$, brokerage $\frac{1}{4}$.
285. \$600 stock sold at 70, brokerage $\frac{1}{3}$.
286. \$1200 stock sold at 80, brokerage $\frac{3}{4}$.
287. \$1000 stock sold at 40, brokerage $\frac{1}{2}$.
288. \$900 stock sold at $90\frac{8}{9}$, brokerage $\frac{5}{9}$.
289. \$1200 stock sold at $60\frac{1}{4}$, brokerage $\frac{1}{4}$.

Ex. **282.** \$640. The net proceeds on one share is \$80; on 8 shares, 8 times \$80, or \$640.

Find the dividend on:

290. \$800 4% stock bought at 90.
291. \$400 2% stock sold at 80.
292. \$500 6% stock sold at $70\frac{1}{4}$.
293. \$600 7% stock bought at 50.
294. \$1200 3% stock sold at $80\frac{1}{8}$.
295. \$1500 5% stock sold at 60.
296. \$2000 9% stock sold at 89.

Ex. **290.** \$32. The dividend on one share is \$4; on 8 shares, 8 times \$4, or \$32.

How many shares may be bought:

297. For \$800 at $79\frac{7}{8}$, brokerage $\frac{1}{8}$?
298. For \$1000 at $49\frac{3}{4}$, brokerage $\frac{1}{4}$?
299. For \$1200 at $59\frac{1}{8}$, brokerage $\frac{7}{8}$?
300. For \$1400 at $69\frac{1}{4}$, brokerage $\frac{3}{4}$?
301. For \$700 at $19\frac{7}{8}$, brokerage $\frac{1}{8}$?
302. For \$2000 at $39\frac{2}{3}$, brokerage $\frac{1}{3}$?
303. For \$3000 at $59\frac{7}{8}$, brokerage $\frac{1}{8}$?

Ex. **297.** 10 shares. The cost of one share is \$80; \$800 will buy as many shares as \$80 is contained times in \$800, or 10 shares.

How much stock gives an income:

304. Of \$200; stock 4%?
305. Of \$400; stock 5%?
306. Of \$120; stock 6%?
307. Of \$100; stock 2%?
308. Of \$900; stock 3%?
309. Of \$8000; stock 4%?
310. Of \$5000; stock 5%?
311. Of \$1800; stock 9%?
312. Of \$2000; stock 10%?
313. Of \$4800; stock 12%?

Ex. **304.** \$5000 stock. The income on one share is \$4; it will take as many shares to yield \$200 as \$4 is contained times in \$200, or 50 shares. 50 shares = \$5000 stock.

What % will I realize on my investment:

314. When 6% stock is bought at 80?
315. When 5% stock is bought at 50?
316. When 8% stock is bought at 40?
317. When 9% stock is bought at 70?
318. When 4% stock is bought at 20?
319. When 3% stock is bought at 60?
320. When 7% stock is bought at 70?

Ex. **314.** $7\frac{1}{2}$%. One share costs \$80 and gains \$6; the gain % is \$6 ÷ \$80, or $7\frac{1}{2}$%.

Find the price of a 4% stock:

321. To equal a 6% stock at 50.
322. To equal a 5% stock at 20.
323. To equal a 7% stock at 60.
324. To equal an 8% stock at 80.
325. To equal a 5% stock at 40.
326. To equal a 12% stock at 80.
327. To equal a 10% stock at 50.

Ex. **321.** \$$33\frac{1}{3}$. One share costs \$50 and gains \$6; \$1 gains $\frac{1}{50}$ of \$6, or 12¢; if \$1 gains 12¢, it will take as many dollars to gain \$4 as 12¢ is contained times in \$4, or \$$33\frac{1}{3}$.

PRACTICAL EXERCISES.

§ 45. At the Lumber Yard.

The same piece of lumber may be called by different names since the classification is not exact. The following may be helpful: —

Lumber — wooden building material.

I. *Boards* — 1 in. thick — less if specified.
   1. Stock boards — boards of uniform width — 12 in. wide.
   2. Fencing — 6 in. wide.
   3. Flooring — matched boards.
   4. Siding or clapboards — $\frac{1}{2}$ in. thick — thicker at one edge.

II. *Dimension Stuff.*
   1. Scantling — 2 in. to 4 in. thick — 3 in. to 4 in. wide.
   2. Joist — 2 in. thick — any width.
   3. Plank — 2 in. thick — wider than 4 in.
   4. Timber — thicker than 2 in. — wider than 4 in.

III. *Foot Stuff* — sold by linear foot.
   1. Battens — for covering cracks.
   2. Moulding — for finishing.

IV. *Laths* — 4 ft. long, $1\frac{1}{2}$ in. wide — 50 to a bunch.

V. *Shingles* — 4 in. wide — 250 to the bunch. They are not of uniform width, but every 4 in. is reckoned as one shingle.

Lumber is sold by the board foot.

A board foot is the equivalent of 1 ft. long, 1 ft. wide, and 1 in. thick.

1 *board ft.*

Inch lumber 12 ft. long contains as many board ft. as there are inches in its width.

For 12 ft. $\times \frac{1}{12}$ ft. $\times$ 1 in. = 1 ft. $\times$ 1 ft. $\times$ 1 in.

How many feet of lumber are there in:

1. 1 8 in., 12 ft. board?
2. 1 8 in., 10 ft. board?
3. 1 8 in., 6 ft. board?
4. 1 8 in., 14 ft. board?
5. 1 8 in., 18 ft. board?
6. 3 6 in., 10 ft. boards?
7. 4 7 in., 16 ft. boards?
8. 1 joist 2 $\times$ 4, 12?
9. 1 joist 2 $\times$ 4, 16?
10. 1 scantling 3 $\times$ 4, 12?
11. 1 plank 2 $\times$ 10, 12?
12. 1 plank 2 $\times$ 10, 16?
13. 4 timbers 4 $\times$ 4, 20?
14. 3 pieces 8 $\times$ 8, 24?
15. 5 pieces 2 $\times$ 4, 12?
16. 8 3 in., 12 ft. boards?
17. 6 14 ft. fencing?
18. 6 pieces 4 $\times$ 6, 20?
* 19. 10 6 in., 12 ft. siding?
20. 10 scantling 3 $\times$ 4, 16?

Ex. **2**. 7. An 8 in. 12 ft. board would contain 8 ft.; $10 = 12 - \frac{1}{6}$ of 12; $8 - \frac{1}{6}$ of $8 = 6\frac{2}{3}$; taking the nearest whole number, 7.

Ex. **7**. 37. 4 7 in. boards = 1 28 in. board. A 28 in. 12 ft. board would contain 28 ft.; $16 = 12 + \frac{1}{3}$ of 12; $28 + \frac{1}{3}$ of $28 = 37\frac{1}{3}$; taking the nearest whole number, 37.

Ex. **9**. 11. This is read, "1 joist 2 by 4, 16." The joist is 2 in. thick, 4 in. wide, 16 ft. long.

Ex. **13**. 107. 4, 4 $\times$ 4 pieces = 1 64 in. board. A 64 in. 12 ft. board would contain 64 ft.; $20 = 12 + \frac{2}{3}$ of 12; $64 + \frac{2}{3}$ of $64 = 106\frac{2}{3}$; taking the nearest whole number, 107.

* Unless otherwise specified, stuff less than an inch thick is counted as an inch thick.

Dealers sometimes use a card like the following, carried out for a great variety of lengths and dimensions. Usually, as in this table, fractions of a foot are neglected.

LUMBER TABLE.

SIZE.	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
2×6	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
2×8	16	19	21	24	27	29	32	35	37	40	43	45	48	51	53
3×4															
4×6															
2×4															
2×3															

21. Verify the results in the 1st horizontal line.
22. Verify the results in the 2d horizontal line.
23. Declare the results for the 3d horizontal line.
24. Declare the results for the 4th horizontal line.
25. Declare the results for the 5th horizontal line.
26. Declare the results for the 6th horizontal line.

How many board feet are there in:

27. 4 plk., 2×4, 12?
28. 2 timbers, 4×6, 40?
29. 9 joists, 2×3, 16?
30. 12 bds., 1×12, 16?
31. 24 fencing 1×6, 16?
32. 4 piece stuff, 3×3, 12?
33. 6 scantling, 2×4, 16?
34. 3 timbers, 6×6, 40?
35. 2 timbers, 8×8, 60?
36. 3 joists, 2×8, 12?
*37. 6 plk., $1\frac{1}{8}$×12, 12?
38. 6 plk., $1\frac{3}{8}$×12, 12?
39. 6 plk., $1\frac{3}{4}$×12, 12?
40. 6 plk., $1\frac{7}{8}$×12, 12?

*Count lumber less than 1 in. thick as 1 in.; from $1\frac{1}{8}$ to $1\frac{1}{4}$, as $1\frac{1}{4}$; from $1\frac{3}{8}$ to $1\frac{1}{2}$, as $1\frac{1}{2}$; from $1\frac{3}{4}$ to 2, as 2.

Laths.

Laths are nailed upon joists $\frac{3}{8}$ of an inch apart to receive the plaster.

41. How many laths are there in a bunch? See p. 131.

42. What are the dimensions of a lath?

43. How many sq. in., including the space between two laths, does 1 lath cover?

44. How many sq. in. will 1 bunch cover?

45. How many sq. in. are there in 3 sq. yd.?

46. How do the results in Exs. 44 and 45 agree with the contractor's rule, "*One bunch of laths will cover 3 sq. yd.*"?

47. Using contractor's rule, how many bunches of laths will be required for the ceiling of a room 18 ft. × 18 ft.?

Shingles.

A bunch of shingles is 20 in. wide and contains 25 courses on each side; a shingle averages 4 in. wide.

48. How many shingles are there in a bunch?

49. If a shingle is laid 4 in. to the weather, how many sq. in. will one shingle cover?

50. How many sq. in. will 1000 shingles cover?

51. How many sq. in. are there in 100 sq. ft.?

52. How do results in Exs. 50 and 51 agree with the carpenter's rule, "*One thousand shingles laid 4 in. to the weather will cover 100 sq. ft.*"?

53. How many bunches of shingles laid 4 in. to the weather will be required for a roof 20 ft. × 40 ft.? Use contractor's rule.

54. How many bunches of shingles laid 4 in. to the weather will be required for a roof 18 ft. × 12 ft.?

§ 46. Measurement of Logs.

By the number of board ft. in a log is meant the number of board ft. in the largest piece of timber that can be sawed from the log.

4 Board ft.

A piece 1 in. × 1 in. × 12 ft. (usually written $1 \times 1, 12$) contains 1 board ft.

A log 12 ft. long contains as many board ft. as there are sq. in. on its squared end, or as many board ft. as there are sq. in. in half the square of its diameter.

If the log is to be sawed into boards, deduct ⅓ for waste.

The area of the greatest inscribed square is $\overline{AB}^2$.

$$\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2.$$

(sq. of hyp. = sum of sqs.)

$$2\,\overline{AB}^2 = \overline{BC}^2.$$

$$\overline{AB}^2 = \frac{\overline{BC}^2}{2}.$$

How many feet of lumber are there in a log:

55. Length 12 ft.; D. 8 in.? **58.** Length 24 ft.; D. 10 in.?
56. Length 16 ft.; D. 10 in.? **59.** Length 32 ft.; D. 18 in.?
57. Length 20 ft.; D. 12 in.? **60.** Length 16 ft.; D. 10 in.?

Ex. **56.** 67 ft. of lumber. $10^2 = 100$; ½ of 100 = 50; if the log were 12 ft. long, there would be 50 board ft.; 50 + ⅓ of 50 = 67.

How many feet of boards are there in a log:

61. Length 12 ft.; D. 8 in.? **64.** Length 20 ft.; D. 6 in.?
62. Length 18 ft.; D. 10 in.? **65.** Length 36 ft.; D. 10 in.?
63. Length 24 ft.; D. 12 in.? **66.** Length 40 ft.; D. 16 in.?

Ex. **61.** 26 ft. There would be 32 ft. of lumber; 32 − ⅛ of 32 = 26.

§ 47. At the Carpet Store.

Carpeting is usually a yard wide, and sold by the linear yard. It is cut up into *breadths*, and these *breadths* are *matched* and sewed together. As the same figure is repeated at intervals varying from 1 inch to 6 feet according to the pattern, few carpets can be matched without loss.

Find the number of breadths and the length of each breadth.

A room 14 ft. by 13 ft. is to be carpeted.

67. How many breadths will be required if they run lengthwise? How many if they run crosswise? *Ans.* 5.

68. If there is no loss in matching, how many yards, 1 yd. wide, should be purchased, the breadths running lengthwise? How many if the breadths run crosswise?

69. How many yards are turned under in each case in Ex. 68?

70. If the same figure is repeated at intervals of 9 in., how many linear inches are wasted on each strip running lengthwise? Explain.

71. If the same figure is repeated at intervals of 9 in., how many linear inches are wasted on each strip running crosswise? Explain.

72. If there are five breadths, on how many is there waste in matching? Explain. *Ans.* 4.

73. What is the cost, at $2 a yard, breadths running lengthwise, the same figures 1 ft. apart, carpet $\frac{3}{4}$ yd. wide? What is the cost, breadths running crosswise?

74. 14 ft. $\times$ 13 ft. $=$ 182 sq. ft.; 182 sq. ft. $= 20\frac{2}{9}$ sq. yd. Will $20\frac{2}{9}$ yd. of carpet 1 yd. wide be sufficient? Why?

§ 48. With the Paper Hanger.

Wall paper is sold by the *double roll*, 48 ft. $\times 1\frac{1}{2}$ ft., or by the *single roll*, 24 ft. $\times 1\frac{1}{2}$ ft. It is cut up into strips, matched, and pasted upon the walls or ceiling.

From the distance around the room in feet, deduct 3 ft. for each opening (door or window). The remainder $\div \frac{3}{2}$ will give the number of strips required for the walls.

The walls and ceiling of an 8 ft. room 20 ft. $\times$ 16 ft. are to be papered; there are four windows and a door.

75. How many strips for the walls will a double roll supply? Explain.

Ans. 6 strips. There is a loss in matching, but this need not be considered, because the paper does not extend to the floor, on account of the *base board*, nor to the ceiling, on account of the border.

76. By the *rule*, how many double rolls will be needed for the wall? Explain. *Ans.* 7.

77. If the strips run lengthwise, how many strips for the ceiling will a double roll supply? Explain. *Ans.* 2.

78. If the strips run crosswise, how many strips for the ceiling will a double roll supply? Explain. *Ans.* 3.

79. How many double rolls will be required for the ceiling, if the strips run lengthwise?

80. How many double rolls will be required for the ceiling, if the strips run crosswise?

81. Do we ever in practice find the exact number of sq. ft. on the walls, deduct for the doors and the windows, and then divide by the number of sq. ft. in a double roll? Why not?

§ 49. AVERAGE.

What is the average of 80, 85, 90, 70, 75, 100, 60, 70? *Ans.* $78\frac{3}{4}$. Their sum is 630; $630 \div 8 = 78\frac{3}{4}$.

A better way is to begin with the first, find the difference between it and each of the others, and unite these differences as we proceed. Thus, $(85 - 80 = 5)$, 5; $(90 - 80 = 10$; add to 5, because 90 is more than 80), 15; $(80 - 70 = 10$; subtract 10, because 70 is less than 80), 5; $(80 - 75 = 5$; subtract), 0; $(100 - 80 = 20$; add), 20; $(80 - 60 = 20$; subtract), 0; $(80 - 70 = 10$; subtract), -10. Since the total of the differences for 8 numbers is -10, the average difference must be $\frac{1}{8}$ of -10, or $-1\frac{1}{4}$; $80 - 1\frac{1}{4} = 78\frac{3}{4}$.

In practice, we say 5, 15, 5, 0, 20, 0, -10, $-1\frac{1}{4}$, $78\frac{3}{4}$.

Find the average of:

82. 80, 70, 70, 60, 90, 84, 85, 87, 90, 93.
83. 85, 86, 87, 88, 90, 92, 93, 94, 95, 96.
84. 70, 75, 80, 85, 90, 95, 100, 90, 80, 70.
85. 75, 80, 82, 84, 78, 85, 71, 60, 54, 96.
86. 50, 80, 85, 70, 75, 100, 99, 86, 87.
87. 76, 68, 54, 47, 97, 98, 96, 97, 95.
88. 84, 83, 80, 87, 89, 88, 82, 81, 85.
89. 66, 67, 52, 53, 47, 84, 87, 91, 90.
90. 100, 100, 90, 95, 98, 99, 97, 100.
91. 75, 76, 73, 50, 62, 64, 70, 80, 89.
92. 91, 89, 94, 93, 91, 99, 100, 50, 60.
93. 89, 89, 88, 87, 86, 93, 95, 96, 97.
94. 45, 37, 29, 50, 49, 61, 62, 59, 48.
95. 84, 64, 94, 74, 54, 100, 90, 80, 70.
96. 65, 70, 80, 85, 90, 94, 86, 85, 72, 84.
97. 90, 92, 92, 93, 93, 94, 94, 94, 96, 98.

Ex. **83.** 90.6. It is best to take as the base some number ending in a cipher; in this take 90. -5, -9, -12, -14, -12, -9, -5, 0, 6; 90.6.

INVOLUTION AND EVOLUTION.

A number written to the right of another, a little above, shows how many times the latter is used as a multiplier.

The number used as a multiplier is the *base;* the number showing how many times the base is used, the *exponent;* the result, the *power;* the process, *involution.*

A number written to the left of another in the symbol $\sqrt{}$, or the denominator of a fractional exponent, calls for the base which taken this number of times as a multiplier will produce the latter.

The result is the *root;* the process, *evolution.*

When the second root, usually called the *square root,* is required, the figure 2 is not written in the $\sqrt{}$.

The third root is usually called the *cube root.*

ILLUSTRATION.

$2^4 = 16$;

read,

2 *to the 4th power* $= 16$.

2, *base;*

4, *exponent;*

16, *power;*

means

$2 \times 2 \times 2 \times 2 = 16$.

$\sqrt[4]{16} = 2$, or $16^{\frac{1}{4}} = 2$.

4 calls for the base which taken 4 times as a multiplier will produce 16;

read,

the 4th root of $16 = 2$.

$\sqrt{16} = 4$;

read,

the square root of $16 = 4$.

$\sqrt[3]{8} = 2$;

read,

the cube root of $8 = 2$.

Declare the value of:

1. 2^5; 3^4; 6^3; 7^3.
2. 8^3; 9^3; 5^3; 4^3.
3. 25^2; 23^2; 22^2; 21^2.
4. 17^2; 18^2; 19^2; 16^2.
5. 3^4; 2^6; 3^5; 2^3.

Declare the value of:

6. $\sqrt{16}$; $\sqrt[3]{27}$; $\sqrt[5]{32}$; $\sqrt[4]{81}$.
7. $16^{\frac{1}{2}}$; $27^{\frac{1}{3}}$; $32^{\frac{1}{5}}$; $81^{\frac{1}{4}}$.
8. $27^{\frac{2}{3}}$; $16^{\frac{3}{4}}$; $64^{\frac{5}{6}}$; $4^{\frac{3}{2}}$.
9. $125^{\frac{2}{3}}$; $64^{\frac{3}{2}}$; $36^{\frac{3}{2}}$; $27^{\frac{5}{3}}$.

Ex. 8., *Ans.* 9. This means extract the cube root of 27, and square the result.

PROPORTION.

Division may be expressed by writing the dividend *before* and the divisor *after* a colon. Such an expression is a *ratio.* See p. 44.

The dividend is the *antecedent;* the divisor, the *consequent.*

Two ratios may be equal. An equality of two ratios is a *proportion.*

The sign of equality is often abbreviated by writing only the extremities of the sign '=,' making : :

The first and last terms of a proportion are *extremes;* the second and third, *means.*

In a proportion, the product of the extremes must equal the product of the means.

If three terms of a proportion are given, the other may be found.

The mean proportional of two quantities is the square root of their product.

ILLUSTRATION.

3 : 4, *ratio;*
read,
3 *is to* 4;
means, $3 \div 4$.
3, *antecedent.*
4, *consequent.*
$3 : 4 = 6 : 8$, *proportion;*
read,
3 *is to* 4 equals 6 *is to* 8.
3 : 4 : : 6 : 8, *proportion;*
read,
3 *is to* 4 as 6 *is to* 8.
3 and 8, *extremes.*
4 and 6, *means.*

$3 : 4 : : 6 : 8.$

$3 \times 8 = 4 \times 6.$

$3 : ( ) : : 6 : 8.$

$( ) = \frac{3 \times 8}{6} = 4.$

$12 : ( ) : : ( ) : 3.$

$( ) = \sqrt{12 \times 3}$
$= 6.$

1. What is the value of the ratio 27 : 81 ? 36 : 72 ? 48 : 144 ?
2. What is the difference between $24 \div 3$ and 27 : 9 ?
3. Which is the greatest, $\frac{3}{4}$, $2 \div 3$, or 1 : 2 ?
4. Find the missing term in the proportion 9 : 18 : : 4 : ( ); in 4 : 12 : : ( ) : 2; in 5 : ( ) : : 7 : 14; in ( ) : 6 : : 8 : 12.
5. What is the mean proportional of 9 and 4 ? 20 and 5 ?

MENSURATION.

§ 50. One Dimension.

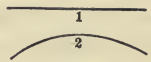
That which has one dimension is a *line.*

A line may extend in the same direction, a *straight* line; or it may constantly change its direction, a *curved* line.

If two straight lines in a plane are extended, they will meet, or they will not meet. If they do not meet, they are *parallel;* if they meet, they form *angles.*

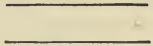
If two lines meet, the angles will be equal, *right* angles, or not equal, *oblique* angles; the larger is *obtuse,* the smaller, *acute.*

A straight line may be parallel to the horizon, a *horizontal* line; perpendicular to the horizon, a *vertical* line; or neither parallel nor perpendicular to the horizon, an *oblique* line.

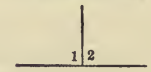


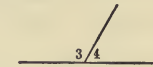
Lines.

1. *Straight.* 2. *Curved.*



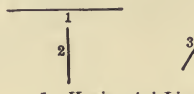
Parallel Lines.





Angles.

1 & 2. *Right.* 3. *Obtuse.*
3 & 4. *Oblique.* 4. *Acute.*



1. *Horizontal Line.*
2. *Vertical Line.*
3. *Oblique Line.*

Define:

1. A line.
2. A straight line.
3. A curved line.
4. Parallel lines.
5. An angle.
6. A right angle.
7. An obtuse angle.
8. An acute angle.
9. Oblique angles.
10. A horizontal line.
11. A vertical line.
12. An oblique line.

§ 51. Two Dimensions.

That which has two dimensions is a *surface*.

Straight lines may inclose a plane surface, a *polygon*.

The least number of straight lines which can inclose a plane is three, a *triangle* (1). The three lines may be equal, an *equilateral* triangle (2); two of them may be equal, *isosceles* triangle (3); or no two of them equal, a *scalene* triangle (4).

A triangle may have one right angle, a *right-angled* triangle (6); one obtuse angle, an *obtuse-angled* triangle (7); or three acute angles, an *acute-angled* triangle (8).

The next number of straight lines which can inclose a plane is four, a *quadrilateral* (9). The quadrilateral may have both pairs of its opposite sides parallel, a *parallelogram* (10); one pair parallel, a *trapezoid* (11); or neither pair parallel, a *trapezium* (12).

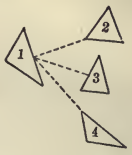
The parallelogram may have its angles right angles, a *rectangle* (13); or not right angles, a *rhomboid* (14).

The rectangle may have its sides all equal, a *square* (15); the rhomboid may have its sides all equal, a *rhombus* (16).

Illustration.

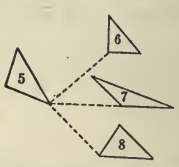
The face of this page is a plane surface, or a *plane*.

All the figures on this page are *polygons*.



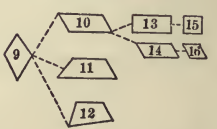
Triangles.

2. *Equilateral.*
3. *Isosceles.*
4. *Scalene.*



Triangles.

6. *Right-angled.*
7. *Obtuse-angled.*
8. *Acute-angled.*



Quadrilaterals.

10. *Parallelogram.*
11. *Trapezoid.*
12. *Trapezium.*
13. *Rectangle.*
14. *Rhomboid.*
15. *Square.*
16. *Rhombus.*

If the angles of a polygon are equal, it is a *regular* polygon.

A regular polygon of five sides is a regular *pentagon* (17); six sides, a regular *hexagon* (18); seven sides, a regular *heptagon*, etc.; infinite number of sides, a *circle*.

ILLUSTRATION.

17. *Regular pentagon.*
18. *Regular hexagon.*
19. *Circle.*

Define:

13. A surface.
14. A polygon.
15. A triangle.
16. An equilateral triangle.
17. An isosceles triangle.
18. A scalene triangle.
19. A right-angled triangle.
20. An obtuse-angled triangle.
21. An acute-angled triangle.
22. A regular polygon.
23. A regular pentagon.
24. A regular hexagon.
25. A regular heptagon.
26. A circle.

27. Beginning with *plane surface* (see note), define *parallelogram, rectangle, rhomboid, rhombus, square.*

28. Beginning with *quadrilateral* (see note), define *parallelogram, rectangle, square, rhombus.*

29. Beginning with *parallelogram* (see note), define *square, rhombus.*

30. Give as short a definition as possible of *square, rhombus.*

NOTE. — A definition may begin with different terms, *e.g.*:

A square is a *plane surface* bounded by two pairs of opposite sides, having each pair parallel, having its angles all right angles, and having its sides all equal. Or,

A square is a *quadrilateral* having its opposite sides parallel, having its angles all right angles, and having its sides all equal. Or,

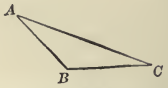
A square is a *parallelogram* having its angles all right angles and having its sides all equal. Or,

A square is a *rectangle* having its sides all equal.

That definition is the best which is the shortest, *provided it begins with a term which is understood by the person for whom the definition is given.*

§ 52. Parts of a Polygon.

That side of a polygon on which it is supposed to rest is its *base;* the distance around a polygon, its *perimeter;* the perimeter of a circle, its *circumference.*

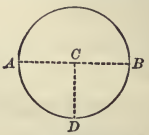


BC, Base.
AB + BC + AC, Perimeter.

In a right-angled triangle, the side opposite the right angle is the *hypotenuse;* the other sides, *legs.*

AB and BC, Legs.
AC, Hypotenuse.

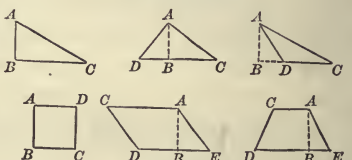
In a circle, a line from the center to the circumference is a *radius;* a line passing through the center and bounded at both extremities by the circumference is a *diameter.*



CD, Radius.
AB, Diameter.

The altitude of a triangle, parallelogram, or trapezoid, is a perpendicular to the base, from the vertex opposite the base.

AB is the altitude in each of these figures. Observe that the base must sometimes be extended.



AB, Altitude.

31. On how many sides may a triangle be supposed to rest?

32. How many bases may it have?

33. How many altitudes may it have?

34. Define the *base* of a polygon.

35. Define the *perimeter* of a polygon; *circumference* of a circle.

36. Define the *radius* of a circle; the *diameter.*

37. Compare the following with the definition of a circle developed on p. 143: A circle is a plane surface bounded by a curved line, every point of which is equally distant from a point within called the center.

§ 53. Rules.

The area of a *parallelogram* is the product of its base and altitude.

The area of a *trapezoid* is half the product of its altitude and the sum of its parallel sides.

The area of a *trapezium* is half the product of its diagonal and the sum of the perpendiculars from the vertices to the diagonal.

The area of a *triangle* is half the product of its base and altitude.

The area of a *triangle* is the square root of the continued product of the half sum of its sides and the remainders found by subtracting each side from the half sum separately.

The square of the *hypotenuse* of a right-angled triangle is equal to the sum of the squares of the other two sides.

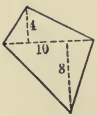
The *circumference* of a circle is twice the radius times 3.1416.

The *area* of a circle is the square of the radius times 3.1416.

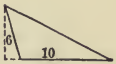
Illustration.

$$\text{Area} = 6 \times 10 = 60 \text{ sq. in.}$$

$$\text{Area} = \frac{8 + 12}{2} \times 6 = 60 \text{ sq. in.}$$



$$\text{Area} = \frac{4 + 8}{2} \times 10 = 60 \text{ sq. in.}$$



$$\text{Area} = \frac{6 \times 10}{2} = 30 \text{ sq. in.}$$

$$\frac{6 + 8 + 10}{2} = 12.$$

$$12 - 6 = 6.$$
$$12 - 8 = 4.$$
$$12 - 10 = 2.$$

$$\text{Area} = \sqrt{12 \times 6 \times 4 \times 2} = 24 \text{ sq. in.}$$

$$5^2 = 3^2 + 4^2 \text{ or, } 25 = 9 + 16.$$

$$\text{Circum.} = 2 \times 10 \times 3.1416$$
$$= 62.832 \text{ in.}$$
$$\text{Area} = 10^2 \times 3.1416 = 314.16 \text{ sq. in.}$$

§ 54. Applications.

Find the area of:

38. A rectangle, B 6 in.; A 4 in.
39. A rhomboid, B 12 in.; A 9 in.
40. A square, B 13 in.
41. A parallelogram, B 14 in.; A 7 in.
42. A trapezoid, ∥ sides 8, 10 in.; A 6 in.
43. A trapezium, D 12 in.; ⊥ 6, 8 in.
44. A triangle, sides 3, 4, 5 in.
45. A triangle, B 6 in.; A 8 in.
46. A rhombus, B 12 in.; A 9 in.
47. A circle, R 5 in.

Find the altitude of:

53. A tri., S 16 sq. in.; B 2 in.
54. A rect., S 40 sq. in.; B 5 in.
55. A trapezoid, S 60 sq. in.; ∥ sides 4, 8 in.

Find the circumference of:

48. A circle, R 2 in.

Find the hypotenuse of:

49. A rt. tri., sides 9, 12 in.
50. A rt. tri., sides 7, 24 in.

Find the other leg of:

51. A rt. tri., hyp. 25 in., A 15 in.
52. A rt. tri., hyp. 13 in., B 12 in.

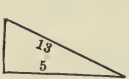
Note. — B, base; A, altitude; ∥, parallel; D, diagonal; ⊥, perpendicular; R, radius; rt. tri., right-angled triangle; rect., rectangle; S, area.

Find the base of:

56. A tri., S 24 sq. in.; A 3 in.
57. A parallelogram, S 32 sq. in.; A 4 in.
58. A square, S 64 sq. in.

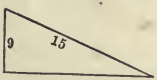
In the following figures find the part indicated.

59.



A = ?

60.

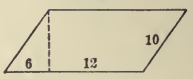


B = ?

61.

Area = ?

62.



Area = ?

63.

R = ?

64.

R = ?

65.

S of circle = ?

66.

Area ring = ?

§ 55. Three Dimensions.

That which has three dimensions is a *solid.*

That part on which a solid rests is its *base;* its other surfaces are *faces;* the union of two faces, an *edge;* the union of three or more edges, a *vertex.*

A solid may have *two* bases equal and parallel polygons, and its faces rectangles, a *prism.* If its bases are triangles, *triangular* prism; squares, *square* prism; . . . circles, *circular* prism or *cylinder.*

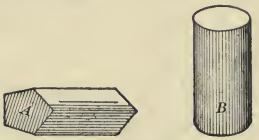
A solid may have *two* bases parallel polygons, and its faces trapezoids, *frustum of a pyramid.* If its bases are triangles, *frustum of triangular pyramid;* . . . circles, *frustum of circular pyramid* or *frustum of cone.*

A solid may have *one* base and its faces triangles, a *pyramid.* If its base is a triangle, *triangular* pyramid; square, *square* pyramid; . . . circle, *circular* pyramid or *cone.*

A solid may have all of its surfaces equal and regular polygons; four triangles, *tetrahedron;* eight triangles, *octahedron;* twenty triangles, *icosahedron;* six squares, *cube;* twelve pentagons, *dodecahedron;* an infinite number of infinitely small polygons, *sphere.*

Illustrations.

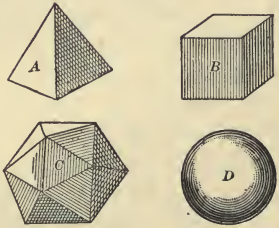
A-BCD, Solid.
BCD, Base.
ACD, Face.
AC, Edge.
A, Vertex.



A, Pentagonal prism.
B, Cylinder.

A, Frustum of hexagonal pyramid
B, Frustum of cone.

A, Pentagonal pyramid.
B, Cone.



A, Tetrahedron.
C, Icosahedron.

B, Cube.
D, Sphere.

§ 56. Rules.

The convex surface of a *prism* or *cylinder* is the product of the perimeter of its base, by its altitude.

The volume of a *prism* or *cylinder* is the product of the area of its base, by its altitude.

The convex surface of a *pyramid* or *cone* is half the product of the perimeter of its base, by its slant height.

The volume of a *pyramid* or *cone* is one third the product of the area of its base, by its altitude.

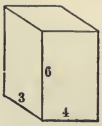
The convex surface of the *frustum* of a *pyramid* or *cone* is half the product of the sum of the perimeters of its two bases, by its slant height.

The volume of the *frustum* of a *pyramid* or *cone* is one third the product of the sum of the areas of its upper base, lower base, and mean proportional base, by its altitude.

The surface of a *sphere* is four times the square of its radius times 3.1416.

The volume of a *sphere* is four thirds times the cube of its radius times 3.1416.

Illustration.

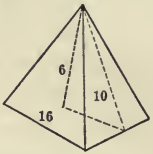


$s=2(3+4)\times 6.$

$v=3\times 4\times 6.$

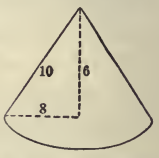
$s=2\times 2\times 3.1416\times 6.$

$v=2^2\times 3.1416\times 6.$



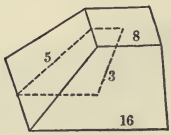
$s=\frac{1}{2}(16+16+16+16)\times 10.$

$v=\frac{1}{3}(16\times 16)\times 6.$



$s=\frac{1}{2}(2\times 8\times 3.1416)\times 10.$

$v=\frac{1}{3}(8\times 8\times 3.1416)\times 6.$

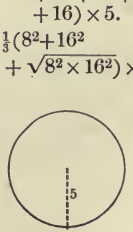


$s=\frac{1}{2}(8+8+8+8+16+16+16+16)\times 5.$

$v=\frac{1}{3}(8^2+16^2+\sqrt{8^2\times 16^2})\times 3.$

$s=\frac{1}{2}(2\times 4\times 3.1416+2\times 8\times 3.1416)\times 5.$

$v=\frac{1}{3}(4^2\times 3.1416+8^2\times 3.1416+\sqrt{4^2\times 8^2\times 3.1416^2})\times 3.$



$s=4\times 5^2\times 3.1416.$

$v=\frac{4}{3}\times 5^3\times 3.1416.$

Define:

67. Solid.
68. Prism.
69. Cylinder.
70. Pyramid.
71. Cone.
72. Frustum of pyramid.
73. Frustum of cone.
74. Regular tetrahedron.
75. Regular octahedron.
76. Regular dodecahedron.
77. Regular icosahedron.
78. Sphere.
79. Triangular prism.
80. Pentagonal pyramid.
81. Frustum of hexagonal pyramid.

State the rule for convex surface of:

82. A prism.
83. A cylinder.
84. A pyramid.
85. A cone.
86. A frustum of a pyramid.
87. A frustum of a cone.
88. A sphere.

State the rule for volume of:

89. A prism.
90. A cylinder.
91. A pyramid.
92. A cone.
93. A frustum of a pyramid.
94. A frustum of a cone.
95. A sphere.

Find the convex surface of:

96. A triangular prism, each side of base 3 in., altitude 12 in.

97. A pentagonal pyramid, each side of base 2 in., slant height 8 in.

98. Frustum of a square pyramid, one side of upper base 5 in., one side of lower base 10 in., slant height 12 in.

99. A sphere, radius 6 in.

Find the volume of:

100. A cylinder, radius of base 5 in., altitude 10 in.

101. A hexagonal pyramid, area of base 36 sq. in., altitude 12 in.

102. Frustum of a cone, radius of upper base 5 in., of lower base 10 in., altitude 12 in.

103. A sphere, radius 3 in.

§ 57. Similarity.

Similar figures must fulfill two conditions:

1st. For every angle of the one there must be an equal angle in the other.

2d. The sides about the equal angles must be in proportion.

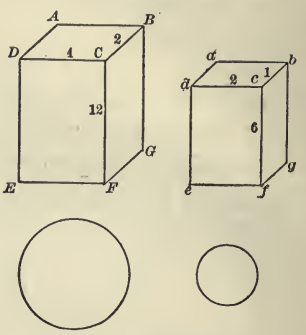
In similar figures:

Linear parts are to each other as homologous linear parts.

Surfaces are to each other as the squares of homologous linear parts.

Volumes are to each other as the cubes of homologous linear parts.

Illustration.



Similar figures.

$CF : cf :: DC : dc.$

$12 : 6 :: 4 : 2.$

Area DF : Area df : : $CF^2 : cf^2$.

Area DF : Area df : : $12^2 : 6^2$.

Vol. AF : Vol. af : : $CF^3 : cf^3$.

Vol. AF : Vol. af : : $12^3 : 6^3$.

104. The radii of two spheres are 4 in. and 2 in. What is the ratio of their circumferences?

$C_1 : C_2 :: 4 : 2.$

$2 : 1$, *Ans.*

105. What is the ratio of their surfaces?

$S_1 : S_2 :: 4^2 : 2^2.$

$4 : 1$, *Ans.*

106. What is the ratio of their volumes?

$V_1 : V_2 :: 4^3 : 2^3.$

$8 : 1$, *Ans.*

107. The circumference of a lead pipe is 6 in.; what is the circumference of a pipe whose diameter is half the diameter of the first?

108. The area of a circle is 10 sq. in.; what is the area of a circle whose diameter is twice the diameter of the first?

109. Two lead pipes are 1 in. and 2 in. in diameter. The area of a horizontal section of the one is how many times a similar section of the other?

110. How many lead pipes 1 in. in diameter will discharge as much water as one pipe 4 in. in diameter?

111. A cannon ball weighs 32 lb.; what is the weight of a similar ball whose diameter is half the diameter of the first?

112. What is the ratio of the surfaces of the two balls in Ex. 111?

113. A is 6 ft. tall; his bronze statue is 12 ft. tall; if the length of A's little finger is $2\frac{1}{2}$ in., what is the length of the little finger of the statue?

114. If it costs \$1 to paint a statue of A's size, what will it cost to paint the statue in Ex. 113?

115. If a statue of A's size weighs 500 lb., what will the statue in Ex. 113 weigh?

116. If a bin 6 ft. deep holds 60 bu., what is the contents of a similar bin 12 ft. deep?

117. If it costs \$10 to make an excavation 6 ft. deep, what is the *approximate* cost of a similar excavation 24 ft. deep?

118. If it costs \$1200 to build a house 20 ft. by 30 ft., what will be the *approximate* cost of a similar house 30 ft. by 45 ft.?

119. If it costs \$16 for material and labor to lay a floor 16 ft. by 20 ft., what will it cost *approximately* to lay a similar floor 20 ft. by 25 ft.?

120. If it costs \$40 to paint a house 30 ft. by 40 ft., what will it cost *approximately* to paint a similar house 45 ft. by 60 ft.?

121. Four pipes each 2 in. in diameter empty into a tank; what must be the diameter of a single pipe to carry away all of the water?

122. A and B bought a ball of twine 8 in. in diameter for \$1; A wound from the outside until the diameter of the ball that was left was 4 in.; what should each pay?

MISCELLANEOUS.

§ 58. Arithmetical Progression.

A series of numbers may increase or decrease by a common difference, *an arithmetical progression.*

The first term is written, a; the last term, l; the number of terms, n; the common difference, d; and the sum of the terms, s.

Every problem may be solved by the formulæ:

$$l = a + (n-1)d \qquad (1)$$

$$s = \frac{n}{2}(a+l) \qquad (2)$$

The arithmetical mean of two numbers is half their sum.

Illustration.

3, 5, 7, 9, 11

14, 11, 8, 5, 3

arithmetical progressions.

Formula (1). The last term equals the first term, plus the number of terms less one times the common difference.

Formula (2). The sum of the terms equals half the number of terms times the sum of the first and last terms.

1. Find l when $a=7$, $n=5$, $d=3$. *Ans.* 19; $l = 7+4\times3=19$.
2. Find s when $n=5$, $a=7$, $l=19$. *Ans.* 65; $s=\frac{5}{2}(7+19)=65$.
3. State the series in Ex. 1. Prove the answer to Ex. 2.
4. Find the arithmetical mean between 7 and 19.
5. Find the sum of the numbers 1 to 25 inclusive.
6. Find the sum of the numbers 1 to 99 inclusive.
7. Translate each formula for arithmetical progression.
8. How far can a man walk in 10 days, going 10 miles the first day and increasing the rate 5 miles per day?

§ 59. Geometrical Progression.

A series of numbers may increase or decrease by a common ratio, *a geometrical progression.*

The first term is written, a; the last term, l; the number of terms, n; the ratio, r; and the sum of the terms, s.

Every problem may be solved by the formulæ:

$$l = ar^{n-1}; \quad (1)$$

$$s = \frac{rl - a}{r - 1}. \quad (2)$$

The geometrical mean of two numbers is the square root of their product.

Illustration.

2, 6, 18, 54
64, 32, 16, 8

geometrical progressions.

Formula (1). The last term equals the first term, times the ratio raised to the power denoted by the number of terms less one.

Formula (2). The sum of the terms equals the quotient, whose dividend is the ratio times the last term, less the first term; and whose divisor is the ratio less one.

9. Find l when $a = 2$, $r = 5$, $n = 3$. *Ans.* 50; $l = 2 \times 5^2 = 50$.

10. Find s when $r = 5$, $l = 50$, $a = 2$. *Ans.* 62; $s = \frac{5 \times 50 - 2}{5 - 1} = 62$.

11. State the series in Ex. 9. Prove the answer to Ex. 10.

12. The extremes are 2 and 250; the ratio is 5; find s.

13. A man bought 6 yards of cloth, giving 2¢ for the first yard, 6¢ for the second, 18¢ for the third, and so on; what did he pay for the last yard? What did he pay for all?

14. What is the geometrical mean between 4 and 25?

15. State the two formulæ for geometrical progression; translate each.

16. A man sold a pair of horses, receiving \$1 for the first shoe, \$2 for the second, \$4 for the third, and so on; the horses being fully shod, how much did he receive?

17. A man bought a pair of oxen, paying 1¢ for the first shoe, 2¢ for the second, 4¢ for the third, and so on; how much did he pay for the last shoe, the oxen being fully shod?

§ 60. Specific Gravity.

The weight of a substance divided by the weight of an equal volume of water is its *specific gravity.*

Approx. Table, S. G.

Gold,	19	Glass,	3	Water,	1
Lead,	11	Stone,	3	Acid,	1.8
Silver,	10	Oak,	.7	Oil,	.9
Iron,	7	Cork,	.2	Air,	.001

A pint is a pound the world round.

A cubic foot of water weighs 62.5 lb.

Illustration.

A vol. lead weighs . . . 22 lb.
Same vol. water weighs 2 lb.

$$\text{S. G. lead} = \frac{22 \text{ lb.}}{2 \text{ lb.}} = 11.$$

Gold is 19 times as heavy as water.

Oak wood is .7 as heavy as water.

Air is .001 as heavy as water.

A pint of *water* weighs a pound (approx.).

18. What is the weight of a pint of gold? Of lead? Of air?

19. What is the weight of a cubic foot of cork?

20. What is the weight of a gallon of water? Of oil? Of acid?

21. What is the weight of a bushel of cork?

22. What is the weight of a cu. ft. of oak wood?

23. Two volumes of lead and 3 of the same volumes of water weigh 25 oz.; what is the specific gravity of lead? Explain.

24. Of a mixture, $\frac{1}{3}$ in volume is oil and $\frac{2}{3}$ water; what is the S. G. of the mixture?

25. What is the S. G. of lead and silver compounded of equal volumes?

26. How many cu. ft. of cork will weigh as much as a cu. ft. of lead?

27. What is the difference in lb. between the weight of a cu. ft. of lead and a cu. ft. of silver?

28. A woman who had learned "A pint is a pound," gave a pint of shot for a lb.; how much did she lose, if shot is 15¢ a lb.?

29. How many gallons of air will weigh one pound?

§ 61. Zero and Infinity.

Numbers may be regarded as existing in three realms.

1. Where their values can be expressed by the decimal notation, *the finite.*

2. Where their values are too great to be expressed by the decimal notation, *the infinite.*

3. Where their values are too small to be expressed by the decimal notation, *the infinitesimal.*

Every number in the greatest realm is expressed by the character ∞.

This does not stand for a single number, but for any one of the countless numbers in this realm.

Every number in the smallest realm is expressed by the character 0.

This does not stand for a single fraction, but for any one of the countless fractions in this realm.

Illustration.

The number of ft. in a mi. can be expressed by the decimal notation.

The number of cu. in. in space is too great to be expressed by the decimal notation.

The difference between 2 and 1.999···, where 9 is repeated without limit, is too small to be expressed by the decimal notation.

One ∞ (infinity) may be 2, 3, or any other number greater or less than another ∞; or twice three times, or any number of times as great.

One 0 (infinitesimal) may be twice, three times, or any number of times as great as another 0.

30. What is the value of $\frac{0}{0}$? *Ans.* Any finite no. as 2, 1000.

31. What is the value of $\frac{\infty}{\infty}$? *Ans.* Any no. as 2, 100, ∞.

32. What is the value of $\frac{0}{6}$? *Ans.* 0.

33. What is the value of $\frac{6}{0}$? *Ans.* ∞. *The smaller the divisor the greater the quotient.*

34. What is the value of $\frac{6}{\infty}$? *Ans.* 0. *The larger the divisor the smaller the quotient.*

GENERAL REVIEW EXERCISES.

1. The sum of eight numbers is 95; the sum of seven of them is 87; what is the eighth number?

2. The addends are 6, 8, 3, 9, 7, 4, 5, 6, 8; what is the sum?

3. The subtrahend is 986; the minuend 1000; what is the remainder?

4. The minuend is 36; the rem. 12; what is the subtrahend?

5. The multiplier is 12; the multiplicand 13; what is the product?

6. The multiplicand is 11; the product 132; what is the multiplier?

7. The dividend is 144; the divisor 18; what is the quotient?

8. The dividend is 119; the quotient 9; the remainder 11; what is the divisor?

9. The dividend is 125; the divisor 16; what is the remainder?

10. The divisor is 9; the quotient 13; the remainder 1; what is the dividend?

11. What number multiplied by 13, with 7 added to the product, will give 85?

12. By what number must 11 be multiplied so that when 4 is taken from the result the remainder will be 128?

13. What is the result when 13 is taken 7 times as an addend?

14. How many times must 12 be taken as an addend to produce 108?

15. Define a *prime* number; numbers *prime to each other*; numbers *severally prime*.

16. Name three composite numbers prime to each other but not severally prime.

17. Give the rule for the divisibility of a number by 2; by 3; by 4; by 5; by 8; by 9; by 11; in general.

18. Name 20 factors of 180180.

19. 77 and 91 are factors of 360360; is their product a factor? Why?

20. Why is 231 exactly contained in 360360?

21. Multiply $5 \times 6 \times 8$ by 7, and express the result by its factors.

22. Divide $27 \times 18 \times 9$ by 3, and express the result by its factors.

23. How many times is $6 \times 8 \times 4 \times 3$ contained in 48×36?

24. How many times is 17×6 contained in $51 \times 2 \times 3$?

25. By an illustration, show that the remainder found by dividing a number by 9, is the same as the remainder found by dividing the sum of its digits by 9.

26. By an illustration, show that the remainder found by subtracting the sum of its digits from a number is divisible by 3.

27. Show that a number is equal to its digit in unit's place, plus ten times its digit in ten's place, plus one hundred times its digit in hundred's place, and so on.

28. State three principles for finding the G. C. D.

29. By the second principle, how can you tell that 4 must be the G. C. D. of 64 and 68?

30. By the third principle, how can you tell that 1 is the G. C. D. of 625 and 1728?

31. State three principles for finding the least common multiple.

32. Find the L. C. M. of 20 and 30; 24 and 36; 30 and 35.

33. Find the L. C. M. of 3, 8, 12, 24, 48, 72. Did you use the third principle?

34. Analyze and explain the meaning of $\frac{5}{8}$ by the first conception; by the second.

35. Which of these two methods was first used to indicate that 5 is to be divided by 8, $5 \div 8$, or $\frac{5}{8}$?

36. $14 \div 3 = 4\frac{2}{3}$. Which is the more natural conception, that 14 divided by 3 equals 4 units and $\frac{2}{3}$ of a unit, or that 14 divided by 3 equals 4, with 2 which is yet to be divided by 3?

37. Change $6\frac{3}{4}$ to an improper fraction; why is this an example in addition of fractions?

38. Divide $17\frac{1}{2}$ by $2\frac{1}{4}$. See p. 63.

39. Divide $17\frac{1}{2}$ by $2\frac{1}{4}$ by inverting the divisor and proceeding as in multiplication. Is this process as easy for mental work as dividing the numerators?

40. What is the difference between $\frac{2}{3}$ of 24 and the number of which 9 is $\frac{3}{4}$?

41. What is $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{3}$ of $6\frac{1}{2}$? Did you find the sum of $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{3}$ before you multiplied by $6\frac{1}{2}$?

42. Reduce $\frac{22}{7}$ to a mixed decimal, and express the result in two ways.

43. State the numeration table for decimals.

44. Express $6 \div 100$ in three ways; what are they?

45. Is there any difference among $\frac{\frac{1}{2}}{100}$, $.00\frac{1}{2}$, and $\frac{1}{2}\%$?

46. Reduce 100 to %.

47. What month of the year is January? Oct.? Dec.? Aug.?

48. How many days in the first six months of a common year?

49. State the number of cu. in. in a gal.; cu. in. in a bu.; relation between cu. ft. and bu.; relation between cu. ft. and gal.

50. How many cu. ft. of hay make a ton? State the relation between cu. ft. corn in the ear and bu. shelled corn.

51. How many drops make 1 teaspoonful? gr. make 1 lb. troy? gr. make 1 lb. apothecaries'? gr. make 1 lb. avoirdupois?

52. How many lb. make 1 bu. oats? 1 bu. corn? 1 bu. potatoes? 1 bu. wheat?

53. In the metric system, state tabulated facts about the unit of long measure; unit of land measure; unit of weight; unit of capacity; unit of wood measure.

54. How many l. in 1 Ml.? How many l. make 1 qt.?

55. How many pints in 3 Hl.? How many l. make 1 cu. m.?

56. How many Ha. in 2 acres?

57. How many cords of wood in 40 steres?

58. Which is the cheaper, to buy meat at 10¢ a lb. or at 20¢ a Kg.? By how much a lb.?

59. What is the sum in lb. of a common English ton, a long English ton, and a metric ton?

60. How much is made per quart by buying chestnuts at $1.60 a bu. and selling at 5¢ a half-pint?

61. How much is gained per lb. by buying salt at $20 a ton and selling at 1¢ an oz.?

62. What is gained on 6 dozen eggs by buying 3 for 2¢ and selling 2 for 3¢?

63. By buying apples at 2 for a cent, and the same number at 3 for a cent, and selling all at 5 for 2¢, I lost 2¢; how many apples did I buy?

64. A man 45 years has a daughter 5 years old. In how many years will she be $\frac{1}{5}$ as old as he? $\frac{1}{3}$ as old? $\frac{1}{2}$ as old? Of the same age?

65. One hunter shot 24 pigeons, another shot 0. The first shot how many times as many as the second?

66. If 8 men will eat a quantity of flour in 15 days, how long will it last if 4 men join them?

67. A loaned $10 and B $15 for the same time and rate; together they received $2 interest; what was the share of each?

68. Three men hired a pasture for $9; A put in 4 horses, B 6, and C 8; what ought each to pay?

69. $\frac{5}{7}$ of a cargo was lost; A, who owned $\frac{1}{3}$ of the whole, lost $100; what was the value of the part that remained?

70. If a man can do $\frac{3}{8}$ of a piece of work in 15 days, how long will it take him to do $\frac{1}{2}$ of it?

71. If a hen and a half lay an egg and a half in a day and a half, how many eggs will 4 hens lay in 3 days?

72. If 3 cats catch 3 rats in 3 minutes, how many cats will be required to catch 100 rats in 100 minutes?

73. A thief bought a pair of boots for $5, and gave in payment a $50 counterfeit bill. The merchant having no money at all, changed the bill at a bank and gave the thief $45 in good money. After the merchant had paid $50 in good money for the bill, what was his entire loss?

74. The freezing and boiling points in the Centigrade thermometer are 0° and 100°; in Fahrenheit's, 32° and 212°. How many degrees C. equal 45° F.?

75. How many degrees F. equal 45° C.?

76. When F. reads 59°, what is the reading of C.?

77. When F. reads 23°, what is the reading of C.?

78. When C. reads 25°, what is the reading of F.?

79. When C. reads 10° below zero, what is the reading of F.?

80. By selling a horse for $36 a man gained $\frac{1}{2}$ of the cost; what was the cost? State this as an example in percentage.

81. By selling a horse for $60 a man lost $\frac{1}{2}$ of the cost; what was the cost? State this as an example in percentage.

82. On an article which cost $24 a merchant gained $33\frac{1}{3}\%$; what would have been the selling price if he had gained half as much?

83. What is the value of $\frac{0}{\infty}$?

84. What is the value of $\frac{\infty}{0}$?

85. What is the value of $0 \times \infty$?

86. May $\frac{0}{0} = 1$? May $\frac{0}{0} = 2$? May $\frac{0}{0} = 100000$? Why?

87. When the diameter of a circle is 0, what is its circumference? What is the ratio of its circumference to its diameter? What is the value of this ratio?

Symbols × and ÷, Page 41

Hazel Valentine Beck

Clorono Baby.

Hazel V. Beck
Miss Olivers Class
Eighth Grade